Solution

QUADRATIC EQUATIONS

Class 11 - Mathematics

1. **(a)** < 0

Explanation:

The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $b^2 - 4ac$ is discriminant. For imaginary roots discriminant should be less than zero.

2.

(b) $i\sqrt{5}, -i\sqrt{5}$

Explanation:

$x^2 + 5 = 0$

$\Rightarrow x^2 = -5$

$\Rightarrow x^2 = 5i^2$

$\Rightarrow x = \pm i\sqrt{5}$ or $i\sqrt{5}, -i\sqrt{5}$

3.

(c) -1

Explanation:

$x^2 + px + q = 0$

$x^2 + qx + p = 0$

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(p - q)x + (q - p) = 0

$\Rightarrow$ (p - q)x = (p - q)

$\Rightarrow$ x = 1

Now, On putting the value of x = 1 in anv one 4 the equation,

We get

1 + p + q = 0

$\Rightarrow$ p + q = -1

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4.

(d) ≥ 0

Explanation:

The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $b^2 - 4ac$ is discriminant, for real roots discriminant should be greater than equal to zero.

5.

(b) p = 1, q = -2

Explanation:

Since, p and q are the roots of the equation

$x^2 + px + q = 0$

So, p + q = -p and pq = q

Now, pq = q $\Rightarrow$ pq - q = 0 $\Rightarrow$ q(p - 1) = 0 $\Rightarrow$ q = 0 and p = 1

If q = 0 then, p = 0 and if p = 1, then, q = -2

6. Sum of roots, $\alpha+\beta=-\frac{5}{2}$
and product of roots, $\alpha\beta=-\frac{4}{2}=-2$
Now, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$ = $\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$
$=\frac{\left(-\frac{5}{2}\right)^2-2(-2)}{-2}=\frac{\frac{25}{4}+4}{-2}=\frac{-41}{8}$
7. Let the side of the smaller square be y and the side of the longer square be x, then
4x - 4y = 24
$\Rightarrow$ x - y = 6
$\Rightarrow$ x = y + 6
According to the question,
$x^2 + y^2 = 468$
$\Rightarrow (y + 6)^2 + y^2 = 468$
$\Rightarrow 2y^2 + 12y + 36 = 468$
$\Rightarrow y^2 + 6y - 216 = 0$
8. Given equation is $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}$ = 0
Comparing it with $ax^2 + bx + c = 0$, we get
a = $\sqrt{3}, b=-\sqrt{2}, c=3\sqrt{3}$
Discriminat = b^2 - 4ac
= $(-\sqrt{2})^2-4\times\sqrt{3}\times3\sqrt{3}$
= 2 - 36 = -34
$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ = $\frac{-(-\sqrt{2})\pm\sqrt{-34}}{2\sqrt{3}}$
= $\frac{\sqrt{2}\pm\sqrt{34}i}{2\sqrt{3}}$
Hence the roots of the given equation are $\frac{\sqrt{2}\pm\sqrt{34}i}{2\sqrt{3}}$.
9. Since, α and β are the roots of $px^2 + qx + 1 = 0$
$\alpha+\beta=\frac{-q}{p};\alpha\beta=\frac{1}{p}$
Now, $\alpha^3\beta^2$ + $\alpha^2\beta^3$ = $\alpha^2\beta^2(\alpha+\beta)$
= $\frac{1}{p^2}\times\left(\frac{-q}{p}\right)=\frac{-q}{p^3}$
10. Since p, q are roots of $3x^2 + 6x + 2 = 0$,
$p+q=-\frac{6}{3}=-2, pq=\frac{2}{3}$
Here, sum (S) = $\frac{p^2}{q}-\frac{q^2}{p}=\frac{-(p^3+q^3)}{pq}$
= $\frac{-\left[(p+q)^3-3pq(p+q)\right]}{pq}$
= $\frac{-\left[(-2)^3-3\left(\frac{2}{3}\right)(-2)\right]}{\frac{2}{3}}$ = $-(-8+4)\cdot\frac{3}{2}$ = 6 and
Product (P) = $\left(-\frac{p^2}{q}\right)\left(-\frac{q^2}{p}\right)=pq=\frac{2}{3}$
Required equation is x^2 - Sx + P = 0,
i.e. $x^2 - 6x + \frac{2}{3} = 0$
i.e, $3x^2 - 18x + 2 = 0$
11. Given, roots of the equation $ax^2 + x + b$ = are real and distinct.
$\therefore$ Discriminant > 0 $\Rightarrow (1)^2 - 4ab > 0$
$\Rightarrow$ 4ab - 1 < 0
Now, $\frac{x^2+1}{x}=4\sqrt{ab}\Rightarrow x+1=4\sqrt{\text{abx}}$ ...(i)
$\Rightarrow x^2-4\sqrt{abx}+1=0$
Its discriminant = $(-4\sqrt{ab})^2$ - 4 $\times$ 1 $\times$ 1
= 16ab - 4 ...(ii)
= 4(4ab - 1) < 0 (from (i))
Thus, the roots of equation (ii) are imaginary (complex)

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Hence, the roots of the equation.

$\frac{x^2+1}{x} = 4\sqrt{ab}$ are imaginary.

12. Let the roots be α, α + 2

Then, $\alpha + \alpha + 2 = \frac{2(p+2)}{p}$

and $\alpha(\alpha + 2) = \frac{3p}{p} = 3$

Now, $\alpha + \alpha + 2 = \frac{(2p+4)}{p} = 2 + \frac{4}{p}$

= $\alpha = \frac{4}{2p} \Rightarrow \alpha = \frac{2}{p}$

$\therefore \alpha(\alpha + 2) = 3 \ \Rightarrow \frac{2}{p}\left(\frac{2}{p} + 2\right) = 3 \ \Rightarrow 3p^2 - 4p - 4 = 0$

$\Rightarrow$ $3p^2$ - 6p +2p - 4=0

$\Rightarrow$ 3p(p - 2) + 2(p -2) = 0

$\Rightarrow$ (3p + 2)(p - 2) = 0 $\Rightarrow p = 2, -\frac{2}{3}$

When p = 2, then $\alpha = \frac{2}{p}$ = 1; so roots are 1, 3

When p = $-\frac{2}{3}$ then, α = $\frac{2}{p}$ = -3; so roots are -3, -1.

13. We have, $2x^2$ - x - 3 = 0

$\Rightarrow$ $2x^2$ - 3x + 2x - 3 = 0

$\Rightarrow$ x(2x - 3) + 1 (2x - 3) = 0

$\Rightarrow$ (x + 1)(2x - 3) = 0

$\Rightarrow$ x = -1 and $x = \frac{3}{2}$

Consider, $\alpha = -1$ and $\beta = \frac{3}{2}$

Now, 2α + 3 = 2(-1) + 3 = 1

and $2\beta + 3 = 2\left(\frac{3}{2}\right) + 3$ = 6

Now the quadratic equation having roots

(2α + 3) and (2β + 3) is

x^2 (2α + 3) + (2β + 3)]x + (2α + 3) + (2β + 3)] = 0

$\Rightarrow$ x^2 - (1 + 6) x + 1 $\times$ 6 = 0

$\Rightarrow$ x^2 - 7x + 6 = 0

14. Since a, b are roots of x^2 + px + 1 = 0,

a + b = -p, ab = 1

Also, c, d are roots of x^2 + qx + 1 = 0

c + d = -q, cd - 1

Now, (a - c) (b - c) + (a - d) (b - d)

= ab - ac - bc + c^2 + ab - ad - bd + d^2

= c^2 + d^2 + 2 - ac - bc - ad - bd [On putting ab = 1]

= c^2 + d^2 + 2cd - (a + b)c - (a + b)d [On putting 1 = cd]

$(c + d)^2$ - (a + b)(c + d) = $(-q)^2$ - (-p) (-q)

= q^2 - pq

Hence Proved

15. We have,

x^2+ lx + m = 0

$\therefore \alpha + \beta = -\frac{1}{1}$ and $\alpha\beta = \frac{m}{1}$

Now, $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

= $-(-1)^2$ - 4(m)

= l^2 - 4m

Now, the quadratic equation having roots

$(\alpha + \beta)^2$ and $(\alpha - \beta)^2$

$x^2 - [(\alpha - \beta)^2 + (\alpha + \beta)^2]x + (\alpha - \beta)^2 (\alpha + \beta)^2 = 0$

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⇒ x^2 - $[(-l)^2 + l^2 - 4 m]$ x + $(-l)^2$ $[l^2 - 4m]$ = 0

⇒ x^2 - $(2l^2 - 4m)$ x + $l^2 - 4m)$ = 0

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