## Conservation of Momentum – Study Notes

### 1. Law of Conservation of Momentum

> **Statement**:
> In the absence of an external force, the total momentum of a system remains constant with time.

Mathematically:

$$F_{\text{ext}} = 0 \Rightarrow \text{Total momentum is conserved}$$

- **Internal forces** do not affect the total momentum of a system.
- This law holds true for **isolated systems** of interacting particles.

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### 2. Derivation: Two-Particle System

Let:

- $m_1, m_2$: Masses of two particles
- $\vec{p}_1, \vec{p}_2$: Their respective momenta
- Net momentum of the system:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 \quad \text{(i)}$$

Let $\vec{F}_1$ and $\vec{F}_2$ be forces acting on particles due to interaction:

- In small interval $\Delta t$, momentum changes:

$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2$$

Dividing by time and taking limit as $\Delta t \to 0$:

$$\frac{d\vec{p}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \vec{F}_1 + \vec{F}_2 \quad \text{(ii)}$$

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### 3. Internal and External Forces

Let:

- $\vec{F}_1 = \vec{F}_{1\text{ext}} + \vec{F}_{1\text{int}}$
- $\vec{F}_2 = \vec{F}_{2\text{ext}} + \vec{F}_{2\text{int}}$

Then:

$$\frac{d\vec{p}}{dt} = (\vec{F}_{1\text{ext}} + \vec{F}_{2\text{ext}}) + (\vec{F}_{1\text{int}} + \vec{F}_{2\text{int}}) \quad \text{(iii)}$$

From **Newton's Third Law**:

$$\vec{F}_{1\text{int}} = -\vec{F}_{2\text{int}} \Rightarrow \vec{F}_{1\text{int}} + \vec{F}_{2\text{int}} = 0$$

So equation (iii) becomes:

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{ext}} \Rightarrow \text{If } \vec{F}_{\text{ext}} = 0, \quad \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{constant}$$

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## 4. Final Result

In an isolated system where $\vec{F}_{\text{ext}} = 0$,
**Total momentum is conserved:**

$$\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$$

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## 5. Applications / Examples

### (i) Bullet-Gun System:

- Bullet fired from a gun.
- Force on bullet = $\vec{F}$, force on gun = $-\vec{F}$
- Both act for the same time $\Delta t$
- By second law:

$$\vec{F}\Delta t = \Delta\vec{p}_{\text{bullet}}, \quad -\vec{F}\Delta t = \Delta\vec{p}_{\text{gun}} \Rightarrow \vec{p}_g + \vec{p}_b = 0 \Rightarrow \vec{p}_g = -\vec{p}_b$$

### (ii) Explosion of a Bomb:

- Bomb at rest → initial momentum = 0
- On explosion into two equal parts flying in opposite directions with equal speed:

$$\vec{p}_1 + \vec{p}_2 = 0$$

- No external force; internal forces only ⇒ total momentum remains zero.

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## Key Points

- Law applies only when **external force is zero**
- **Internal forces cancel out** due to Newton's third law
- Conservation holds in all types of interactions: collisions, explosions, etc.

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