5. Transformations of Quadratic Equations

1. Changing the Roots by a Fixed Number

If α, β are roots of $ax^2 + bx + c = 0$ and each root is increased by h,
New roots: $\alpha + h,\ \beta + h$

New equation:
Let $y = x - h$, substitute into original equation, and simplify.

Example 1:
Given $x^2 - 5x + 6 = 0$, increase each root by 2.
Original roots: 2, 3 → New roots: 4, 5
Equation: $(x-4)(x-5) = 0 \rightarrow x^2 - 9x + 20 = 0$

2. Changing the Roots to their Reciprocals

If α, β are roots, new roots are $\frac{1}{\alpha}, \frac{1}{\beta}$.

New equation:

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$$

Using relations:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

We get:

$$x^2 - \frac{\alpha+\beta}{\alpha\beta}x + \frac{1}{\alpha\beta} = 0$$

Example 2:
Given $2x^2 - 5x + 3 = 0$
Sum = $\frac{5}{2}$, Product = $\frac{3}{2}$
New equation: $x^2 - \frac{\frac{5}{2}}{\frac{3}{2}}x + \frac{1}{\frac{3}{2}} = 0$

$$x^2 - \frac{5}{3}x + \frac{2}{3} = 0$$

3. Roots Changed to their Negatives

If α, β are roots, new roots are $-\alpha, -\beta$.

New equation:
Replace x by $-x$ in the original equation.

Example 3:
Given $x^2 - 4x + 3 = 0$, new equation with roots $-\alpha, -\beta$:
Replace x by $-x$: $(-x)^2 - 4(-x) + 3 = 0$

$$x^2 + 4x + 3 = 0$$

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4. Roots Multiplied by a Constant k

If α, β are roots, new roots are $k\alpha, k\beta$.

New equation:
Replace x by $\frac{x}{k}$ in the original equation.

Example 4:
Given $x^2 - 7x + 10 = 0$, roots multiplied by 3:
Replace x with $\frac{x}{3}$:

$$\left(\frac{x}{3}\right)^2 - 7\left(\frac{x}{3}\right) + 10 = 0 \Rightarrow \ x^2 - 21x + 90 = 0$$

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5. Equation with Sum or Product of Roots Given

If sum of roots = S, product = P:
Equation: $x^2 - Sx + P = 0$

Example 5:
Sum = 9, Product = 14 → Equation: $x^2 - 9x + 14 = 0$
