9. Special Types of Quadratic Equations

1. Pure Quadratic Equations

- Form: $ax^2 + c = 0$ (i.e., $b = 0$)
- Solved by isolating x^2 and taking square roots.

Example 1:
$3x^2 - 12 = 0$
$x^2 = 4 \rightarrow x = \pm 2$

2. Quadratic Equations with $c = 0$

- Form: $ax^2 + bx = 0$
- Factor out x: $x(ax + b) = 0 \rightarrow x = 0$ or $x = -\frac{b}{a}$.

Example 2:
$x^2 - 7x = 0 \rightarrow x(x - 7) = 0 \rightarrow x = 0, 7$

3. Perfect Square Quadratic Equations

- Discriminant $D = 0 \rightarrow$ equal roots.

Example 3:
$x^2 - 6x + 9 = 0 \rightarrow (x - 3)^2 = 0 \rightarrow x = 3$ (double root)

4. Equations with Complex Coefficients

- Coefficients may be complex numbers.
- Roots found using quadratic formula, following complex number arithmetic.

Example 4:
$x^2 + (2 + i)x + (1 + i) = 0$
Use:

$$x = \frac{-(2 + i) \pm \sqrt{(2 + i)^2 - 4(1 + i)}}{2}$$

5. Quadratic in Form

- Equations that look higher degree but are reducible to quadratic in x^n.

Example 5:
Solve $2x^4 - 5x^2 + 3 = 0$.
Let $y = x^2$: $2y^2 - 5y + 3 = 0$
Factor: $(2y - 3)(y - 1) = 0 \rightarrow y = \frac{3}{2}$ or 1
Roots: $x = \pm\sqrt{\frac{3}{2}},\ \pm 1$

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6. Reciprocal Quadratic Equations

- If replacing x by $\frac{1}{x}$ leaves the equation unchanged, it is reciprocal.
- Roots come in reciprocal pairs.

Example 6:
$2x^2 + 3x + 2 = 0$ → Coefficients symmetric → reciprocal roots.
