Lecture Notes – Roots of a Quadratic Equation (Class 11 – ISC)

2. Roots of a Quadratic Equation

Definition

The **roots** (or solutions) of a quadratic equation are the values of x that satisfy:

$$ax^2 + bx + c = 0, \quad a \neq 0$$

Each quadratic equation has **two roots**, which may be:

- Real and distinct
- Real and equal
- Complex conjugates

Relation between Roots and Coefficients

If α and β are the roots of $ax^2 + bx + c = 0$, then:

$$\alpha + \beta = -\frac{b}{a} \quad \text{(Sum of roots)}$$

$$\alpha\beta = \frac{c}{a} \quad \text{(Product of roots)}$$

Forming a Quadratic Equation from Given Roots

If roots are α and β:

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Nature of Roots and Discriminant

The **discriminant** $D = b^2 - 4ac$ determines the nature of roots:

- $D > 0$: Real and distinct
- $D = 0$: Real and equal
- $D < 0$: Complex conjugates

Model Problems

Example 1: Find the roots of $x^2 - 5x + 6 = 0$
Factorizing: $(x - 2)(x - 3) = 0$
Roots: $x = 2,\ x = 3$
Sum = 5, Product = 6

Example 2: Form the quadratic equation whose roots are 4 and -3
Sum = $4 + (-3) = 1$
Product = $4 \times (-3) = -12$
Equation:

$$x^2 - (1)x + (-12) = 0 \quad \Rightarrow \quad x^2 - x - 12 = 0$$

Example 3: Determine the nature of roots of $2x^2 - 3x + 4 = 0$
$D = (-3)^2 - 4(2)(4) = 9 - 32 = -23 < 0$
Conclusion: Roots are complex conjugates.

Example 4: The sum of roots is 8 and the product of roots is 15. Find the equation.
Equation:

$$x^2 - (8)x + 15 = 0$$
