Lecture Notes – Properties and Symmetry in Quadratic Equations (Class 11 – ISC)

6. Properties and Symmetry in Quadratic Equations

1. Symmetric Functions of Roots

If α, β are roots of $ax^2 + bx + c = 0$:

- **Sum of roots:** $\alpha + \beta = -\frac{b}{a}$
- **Product of roots:** $\alpha\beta = \frac{c}{a}$
- Any symmetric function in α, β can be expressed in terms of $\alpha + \beta$ and $\alpha\beta$.

Example 1:
Find $\alpha^2 + \beta^2$ given $\alpha + \beta = 5$ and $\alpha\beta = 6$.

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 25 - 12 = 13$$

2. Condition for Common Roots in Two Quadratic Equations

Given:

$$a_1x^2 + b_1x + c_1 = 0$$

$$a_2x^2 + b_2x + c_2 = 0$$

They have a **common root** if:

$$\frac{b_1}{a_1} - \frac{b_2}{a_2} \text{ and } \frac{c_1}{a_1} - \frac{c_2}{a_2}$$

are in the same ratio (elimination method).

Example 2:
Check if $x^2 - 5x + 6 = 0$ and $2x^2 - 9x + 10 = 0$ have a common root.
Subtract equations after multiplying appropriately → root = 2 is common.

3. Condition for Roots to be Equal in Magnitude but Opposite in Sign

If roots are α and $-\alpha$:

- Sum = 0 → $b = 0$
- Product = $-\alpha^2 = \frac{c}{a} < 0$

Example 3:
$x^2 - 9 = 0$ → roots $3, -3$ satisfy the condition.

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4. Condition for Roots to be Reciprocals

If roots are α and $\frac{1}{\alpha}$:

- Product = 1 → $\frac{c}{a} = 1$

Example 4:
$2x^2 - 5x + 2 = 0$ → Product = $\frac{2}{2} = 1$ → reciprocals.

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5. Condition for Both Roots to be Positive

For $ax^2 + bx + c = 0$ with $a > 0$:

- Sum > 0 → $-\frac{b}{a} > 0$
- Product > 0 → $\frac{c}{a} > 0$

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6. Condition for Both Roots to be Negative

For $a > 0$:

- Sum < 0 → $-\frac{b}{a} < 0$
- Product > 0 → $\frac{c}{a} > 0$

Model Problems

Example 5:
Find the condition that both roots of $3x^2 + kx + 5 = 0$ are positive.
Conditions:

$$-\frac{k}{3} > 0 \quad \Rightarrow \quad k < 0$$

$$\frac{5}{3} > 0 \quad \Rightarrow \quad \text{True for all } k$$

Thus $k < 0$.

Example 6:
Find the quadratic equation whose roots are squares of the roots of $x^2 - 5x + 6 = 0$.
Original roots: 2, 3 → Squares: 4, 9
New equation: $(x - 4)(x - 9) = 0$ → $x^2 - 13x + 36 = 0$
