

Newton's laws lecture notes

Newton's Second Law in Component Form - Detailed Explanation

1. Vector Form of Newton's Second Law:

Newton's Second Law of Motion in vector form is:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

For constant mass m , momentum $\vec{p} = m\vec{v}$, so:

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

2. Component Representation of Vectors:

Any vector in 3D space can be broken into components along the x, y, and z directions:

- **Force vector:**

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

- **Momentum vector:**

$$\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$$

- **Acceleration vector:**

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

3. Applying Newton's Second Law:

From the vector law:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Substitute vector components:

$$\begin{aligned} F_x\hat{i} + F_y\hat{j} + F_z\hat{k} &= \frac{d}{dt}(p_x\hat{i} + p_y\hat{j} + p_z\hat{k}) \\ &= \frac{dp_x}{dt}\hat{i} + \frac{dp_y}{dt}\hat{j} + \frac{dp_z}{dt}\hat{k} \end{aligned}$$

For constant mass:

$$p_x = mv_x, \quad p_y = mv_y, \quad p_z = mv_z \Rightarrow \frac{dp_x}{dt} = ma_x, \text{ etc.}$$

4. Component Form Equations:

By comparing components along each axis, we get:

- In the **x-direction**:

$$F_x = \frac{dp_x}{dt} = ma_x$$

- In the **y-direction**:

$$F_y = \frac{dp_y}{dt} = ma_y$$

- In the **z-direction**:

$$F_z = \frac{dp_z}{dt} = ma_z$$

5. Significance of Component Form:

- **Simplifies vector problems:** Allows solving motion equations axis by axis.
 - **Useful in multiple dimensions:** Helps analyze 2D and 3D motion.
 - **Real-world application:** Used in physics simulations, engineering designs, and mechanics problems.
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