3. Methods of Solving Quadratic Equations

Method 1 – Factorization Method

- Express $ax^2 + bx + c = 0$ as a product of two binomial factors.
- Set each factor equal to zero to get the roots.

Example 1:
Solve $x^2 - 5x + 6 = 0$
Factorize: $(x - 2)(x - 3) = 0$
Roots: $x = 2,\ x = 3$

Method 2 – Completing the Square

- Transform the quadratic into a perfect square form.

Example 2:
Solve $x^2 + 6x + 5 = 0$
Step 1: $x^2 + 6x = -5$
Step 2: Add $\left(\frac{6}{2}\right)^2 = 9$ to both sides:
$x^2 + 6x + 9 = 4$
Step 3: $(x + 3)^2 = 4$
Step 4: $x + 3 = \pm 2 \to x = -1, -5$

Method 3 – Quadratic Formula

The roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 3:
Solve $2x^2 - 3x + 1 = 0$
$a = 2,\ b = -3,\ c = 1$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(1)}}{2(2)} = \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm 1}{4}$$

Roots: $x = 1,\ \frac{1}{2}$

Method 4 – Using Sum and Product of Roots

When $\alpha + \beta$ and $\alpha\beta$ are known,

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Example 4:
Sum = 5, Product = 6
Equation: $x^2 - 5x + 6 = 0$
Roots: $x = 2, 3$

Method 5 – Special Cases

- **When $b = 0$**: Equation reduces to $ax^2 + c = 0$
  Example: $3x^2 - 12 = 0 \rightarrow x^2 = 4 \rightarrow x = \pm 2$

- **When $c = 0$**: Equation reduces to $ax^2 + bx = 0$
  Example: $x^2 - 5x = 0 \rightarrow x(x - 5) = 0 \rightarrow x = 0, 5$