

7. Graphical Interpretation of Quadratic Equations

1. Graph Shape

- The graph of $y = ax^2 + bx + c$ is a **parabola**.
 - **Opening direction:**
 - $a > 0 \rightarrow$ opens **upwards**
 - $a < 0 \rightarrow$ opens **downwards**
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2. Axis of Symmetry

- The parabola is symmetric about the vertical line:

$$x = -\frac{b}{2a}$$

3. Vertex of the Parabola

- Vertex coordinates:

$$\left(-\frac{b}{2a}, \frac{-D}{4a} \right)$$

where $D = b^2 - 4ac$.

4. Intercepts

- **Y-intercept:** At $x = 0, y = c$.
 - **X-intercepts:** Points where $y = 0$, found by solving $ax^2 + bx + c = 0$.
 - If $D > 0 \rightarrow$ two intercepts
 - If $D = 0 \rightarrow$ one intercept (vertex touches x-axis)
 - If $D < 0 \rightarrow$ no real x-intercepts
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5. Effect of Coefficients

- a : Controls **opening direction** and **width** of parabola.

- Large $|a|$ → narrow curve
 - Small $|a|$ → wide curve
 - b : Affects the **horizontal position** of the vertex.
 - c : Determines **y-intercept**.
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Model Problems

Example 1: Sketch $y = x^2 - 4x + 3$

- $a = 1, b = -4, c = 3$
 - Axis: $x = \frac{4}{2} = 2$
 - Vertex: $(2, y(2)) = (2, -1)$
 - Discriminant: $16 - 12 = 4 > 0$ → two x-intercepts
 - Factor: $(x - 1)(x - 3) = 0$ → x-intercepts at $x = 1, 3$
 - Shape: Opens upward, vertex at $(2, -1)$
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Example 2: Sketch $y = -2x^2 + 4x - 1$

- $a = -2, b = 4, c = -1$
 - Axis: $x = \frac{-4}{-4} = 1$
 - Vertex: $(1, y(1)) = (1, 1)$
 - Discriminant: $16 - 8 = 8 > 0$ → two x-intercepts
 - Shape: Opens downward, vertex at $(1, 1)$
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Example 3: $y = x^2 + 4x + 5$

- $D = 16 - 20 = -4 < 0$ → no x-intercepts
 - Vertex: $(-2, 1)$, opens upward
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