Lecture Notes – Discriminant and Nature of Roots (Class 11 – ISC)

4. Discriminant and Nature of Roots

Definition of Discriminant

For a quadratic equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

the **discriminant** is defined as:

$$D = b^2 - 4ac$$

It plays a key role in determining the **nature of the roots**.

Nature of Roots Based on D

1. $D > 0$ **and perfect square** → Roots are **real, distinct, and rational**
2. $D > 0$ **but not a perfect square** → Roots are **real, distinct, and irrational**
3. $D = 0$ → Roots are **real and equal** (repeated roots)
4. $D < 0$ → Roots are **non-real complex conjugates**

$$\alpha = p + iq, \quad \beta = p - iq$$

Geometrical Interpretation

For $y = ax^2 + bx + c$:

- **Two points of intersection** with the x-axis → $D > 0$
- **One point of tangency** with the x-axis → $D = 0$
- **No intersection** with the x-axis → $D < 0$

Model Problems

Example 1: Determine the nature of roots of $3x^2 - 5x + 2 = 0$

$$D = (-5)^2 - 4(3)(2) = 25 - 24 = 1 > 0$$

Roots are **real, distinct, and rational**.

Example 2: Determine the nature of roots of $2x^2 - 4x + 2 = 0$

$$D = (-4)^2 - 4(2)(2) = 16 - 16 = 0$$

Roots are **real and equal**.

Example 3: Determine the nature of roots of $x^2 + 2x + 5 = 0$

$$D = (2)^2 - 4(1)(5) = 4 - 20 = -16 < 0$$

Roots are **complex conjugates**: $-1 \pm 2i$

Example 4: Solve $5x^2 - 6x + 1 = 0$ and classify roots

$$D = (-6)^2 - 4(5)(1) = 36 - 20 = 16 > 0$$

Roots:

$$x = \frac{6 \pm 4}{10} \quad \Rightarrow \quad x = 1,\ \frac{1}{5}$$

Nature: **real, distinct, rational**.
