8. Applications of Quadratic Equations

1. Word Problems in Motion

- **Example 1:**
  A ball is thrown upward with velocity $20\,m/s$ from the ground. The height after t seconds is given by:

$$h = 20t - 5t^2$$

Find when it hits the ground.
Solution: $h = 0 \rightarrow 20t - 5t^2 = 0$
$t(20 - 5t) = 0 \rightarrow t = 0$ or $t = 4$ seconds.

2. Geometry-Based Problems

- **Example 2:**
  The length of a rectangle is 3 m more than its breadth. The area is 70 m². Find dimensions.
  Let breadth = x, length = $x + 3$.
  Area: $x(x + 3) = 70$

$$x^2 + 3x - 70 = 0$$

Factor: $(x + 10)(x - 7) = 0 \rightarrow x = 7$ (positive value)
Length = 10 m, Breadth = 7 m.

3. Age Problems

- **Example 3:**
  A father is 4 times as old as his son. In 5 years, he will be 3 times as old as his son.
  Let son's age = x, father's age = $4x$.
  After 5 years:

$$4x + 5 = 3(x + 5)$$

$$4x + 5 = 3x + 15$$

$$x = 10$$

Father's age = 40 years.

4. Maximum and Minimum Values

For $y = ax^2 + bx + c$:

- If $a > 0$: Minimum at vertex $x = -\frac{b}{2a}$
- If $a < 0$: Maximum at vertex $x = -\frac{b}{2a}$

Example 4:
Find maximum value of $y = -x^2 + 6x + 5$.
Vertex: $x = -\frac{6}{-2} = 3$
Max value: $y(3) = -(9) + 18 + 5 = 14$.

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5. Problems Reducible to Quadratic Form

Some equations are not quadratic in appearance but can be transformed.

Example 5:
Solve $x^4 - 5x^2 + 4 = 0$.
Let $y = x^2 \rightarrow y^2 - 5y + 4 = 0$
Factor: $(y - 4)(y - 1) = 0 \rightarrow y = 4$ or $y = 1$
Thus $x = \pm 2,\ \pm 1$.
