

Vector Algebra Resources

ChatGPT said:



Here is a concise summary of important vector concepts and their associated formulae:

1. Vector Basics:

- **Magnitude of a Vector:**

$$|\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

where A_x, A_y, A_z are the components of vector \mathbf{A} .

- **Unit Vector:**

A unit vector in the direction of \mathbf{A} is:

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{|\mathbf{A}|}$$

2. Vector Addition:

- **Addition of Two Vectors:**

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors in the x, y, and z directions.

3. Scalar (Dot) Product:

- The scalar product of two vectors \mathbf{A} and \mathbf{B} is:

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

where θ is the angle between the two vectors.

4. Vector (Cross) Product:

- The vector product of two vectors \mathbf{A} and \mathbf{B} is:

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}| \sin \theta \hat{n}$$

where \hat{n} is the unit vector perpendicular to both \mathbf{A} and \mathbf{B} , and:

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

5. Projection of a Vector:

- The projection of vector \mathbf{A} on \mathbf{B} is:

$$\text{Proj}_{\mathbf{B}}\mathbf{A} = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{B}|}$$

6. Angle Between Two Vectors:

- The angle θ between two vectors \mathbf{A} and \mathbf{B} is given by:

$$\cos \theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$$

7. Scalar Triple Product:

- The scalar triple product of three vectors \mathbf{A} , \mathbf{B} , \mathbf{C} is:

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \text{Volume of parallelepiped}$$

This gives the volume of the parallelepiped formed by the three vectors.

8. Vector Triple Product:

- The vector triple product of three vectors \mathbf{A} , \mathbf{B} , \mathbf{C} is:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

9. Position Vector and Distance:

- The position vector of a point $P(x_1, y_1, z_1)$ is:

$$\mathbf{r}_P = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

- The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

10. Equation of a Line (Vector Form):

- The vector equation of a line passing through point \mathbf{A} and parallel to vector \mathbf{B} is:

$$\mathbf{r} = \mathbf{A} + \lambda\mathbf{B}$$

where λ is a scalar parameter.

This set of formulae and concepts forms the core of vector algebra used in physics and mathematics. These are widely applicable in various problems involving vectors.