Trigonometry ISC Topics

Given expression:

$$(\csc\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$$

Step 1: Use reciprocal identities

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

Substitute into the expression:

$$\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\tan\theta + \frac{1}{\tan\theta}\right)$$

Step 2: Simplify each term

First term:

$$\frac{1 - \sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

Second term:

$$\frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

Third term:

$$\tan\theta + \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

Step 3: Multiply all terms

$$\left(\frac{\cos^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta\cos\theta}\right)$$

Combine terms:

$$\frac{\cos^2\theta \cdot \sin^2\theta}{\sin\theta \cdot \cos\theta \cdot \sin\theta \cdot \cos\theta} = \frac{\cos^2\theta \cdot \sin^2\theta}{\cos^2\theta \cdot \sin^2\theta} = 1$$

Final Answer:

$$\boxed{1}$$