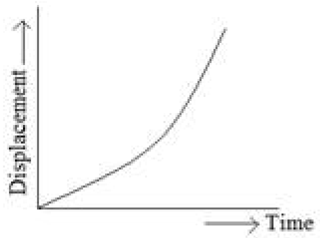
Solution

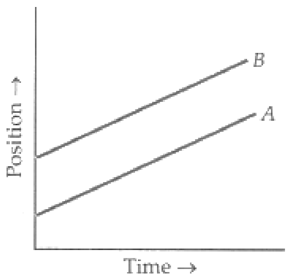
MOTION IN A STRAIGHT LINE

Class 11 - Physics

1. We know that, $v = \frac{dx}{dt}$
   On differentiating w.r.t. t, we get
   $v = \frac{d}{dt}\left(a + bt^2\right)$ = 2bt = 5t m/s [$\because$ b = 2.5 m]
   At t = 0, v = 0, t =2s and v = 10 m/s
2. No, if the speed is zero, the velocity will be necessarily zero.
3. Acceleration
4. Yes, a particle in uniform circular motion has a constant speed but varying velocity because of the change in its direction of motion at every point.
5. Yes. A body thrown vertically upwards has zero velocity at its highest point but has acceleration equal to the acceleration due to gravity.
6. No, speed of an object can never be negative because distance is also always positive.
7. The displacement time graph for a uniformly accelerated motion is shown below:



   The graph is parabolic in shape.
8. As the relative velocity is zero, the two bodies A and B have equal velocities. Hence their position-time graphs are parallel straight lines, equally inclined to the time-axis as shown in Figure.



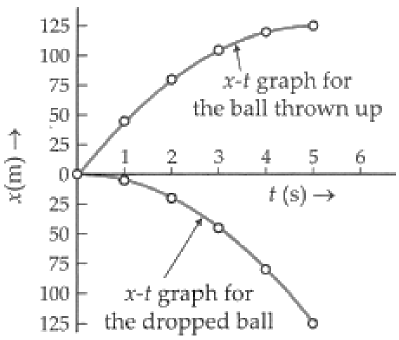
9. Using, $s = ut + \frac{1}{2}at^2$, we have
   $-h = 19.6 \times 6 + \frac{1}{2} \times (-9.8) \times 6^2$
   h = 58.8 m
10. Yes. s = $\left(\frac{u+v}{2}\right) \times t = \left(\frac{v - at + v}{2}\right) \times t = vt - \frac{1}{2}at^2$
11. Given: Total distance covered, S = 100 m, u = 0
    For first $\frac{3}{4}$th of the run, distance $S_1 = \frac{3}{4}S = \frac{3}{4} \times 100$ = 75 m, a = +1 ms^{-2}. Let the time for this part of the run be t_1, then using the second equation of motion,
    $S_1 = ut_1 + \frac{1}{2}at_1^2$
    $75 = 0 + \frac{1}{2} \times 1 \times t_1^2$ or $t_1^2 = 75 \times 2 = 150$
    $\Rightarrow t_1 = \sqrt{150}$ = 12.25 s
    and final velocity v = u + at_1 = 0 + 1$\times$12.25 = 12.25 m s^{-1}.
    For remaining $\frac{1}{4}$ th run, distance $S_2 = S - S_1$ = 100 - 75 = 25 m, uniform velocity v = 12.25 m s^{-1}
    $\therefore$ Time for this run $t_2 = \frac{S_2}{v} = \frac{25}{12.25}$ = 2.04 s
    Thus, total time taken by the sprinter to complete the race is, t = $t_1 + t_2$ = 12.25 s + 2.04 s = 14.29 s = 14.3 s.
12. Let h be the height of the cliff and n be the total time taken by the stone while falling. As
    u = 0

$a = g = 9.8 m/s^2$
$S_{nth} = u + \frac{a}{2}(2n-1)$
$44.1 = 0 + \frac{9.8}{2}(2n-1)$
$88.2 = 9.8\,(2n-1)$
$2n - 1 = 9$
$n = \frac{10}{2} = 5s$
for Height of the cliff using equation
$h = ut + \frac{1}{2}at^2$
$h = un + \frac{1}{2}gn^2$
$h = 0 \times 5 + \frac{1}{2} \times 9.8 \times (5)^2$
$h = 4.9 \times 25$
$h = 122.5\ m$

13. a. The given x-t graph, shown in (a), does not represent one-dimensional motion of the particle. This is because a particle cannot have two positions at the same instant of time.
    b. The given v-t graph, shown in (b), does not represent one-dimensional motion of the particle. This is because a particle can never have two values of velocity at the same instant of time.
    c. The given v-t graph, shown in (c), does not represent one-dimensional motion of the particle. This is because speed being a scalar quantity cannot be negative.
    d. The given v-t graph, shown in (d), does not represent one-dimensional motion of the particle. This is because the total path length travelled by the particle cannot decrease with time.
14. We can find the distances covered by the two balls at different instants of time by using the formula,
    s = ut + $\frac{1}{2}gt^2$

Time	For the ball dropped down u = 0, g = + 10 ms^{-2}	For the ball thrown up u = 50 ms^{-1}, g = - 10 ms^{-2}
	s = $5t^2$	s = 50t - $5t^2$
t = 1 s	s = 5 m	s = 45 m
t = 2 s	s = 20 m	s = 80 m
t = 3 s	s = 45 m	s = 105 m
t = 4 s	s = 80 m	s = 120 m
t = 5 s	s = 125 m	s = 125 m

The position-time graphs for the motion of the two balls are as shown in Fig.



15. Let us consider function of motion
    $x(t) = A + Be^{-\gamma t}$ …(i)
    Where $\gamma$and A, is a constant B is amplitude
    x (t) is displacement at time t, where A > B andγ>0
    $v(t) = \frac{dx(t)}{dt} = 0 + (-\gamma)Be^{-\gamma t} = -\gamma Be^{-\gamma t}$
    $a(t) = \frac{d}{dt}[v(t)] = \frac{d}{dt}(-\gamma B \exp^{-\gamma t}) = \left(\gamma^2 B \exp^{-\gamma t}\right)$
    From (i) ∴A > B so x is always + ve i.e., x > 0.
    From (ii) v is always negative from (ii) v < 0
    From (iii) a is always again positive a >0

As the value of $\gamma^2 Be^{-\gamma}$ can varies from 0 to $+\infty$

16. i. Distance travelled by the particle between 0 sec and 10 sec is
s = Area under speed-time graph = $\frac{1}{2}$(10 - 0) (12 - 0) = 60 m
ii. Average speed = $\frac{\text{Total distance travelled}}{\text{Time taken}}$
= $\frac{60}{10}$ = 6 ms^{-1}
iii. Speed is minimum at t = 0 s and f = 10 s
iv. Speed is maximum at t = 5

17. a. Distance traveled by the particle = Area under the given graph
= $\frac{1}{2}\times$ (10 - 0) $\times$ (12 - 0) = 60 m
Average speed = $= \frac{\text{Distance}}{\text{Time}} = \frac{60}{10} = 6\text{m/s}$
b. Let s_1 and s_2 be the distances covered by the particle between time t = 2 s to 5 s and t = 5 s to 6 s respectively.
Total distance (s) covered by the particle in time t = 2 s to 6 s
s = s_1 + s_2 … (i)
For distance s_1:
Let 'u' be the velocity of the particle after 2 s and 'a' be the acceleration of the particle in t = 0 to t = 5 s.
Since the particle undergoes uniform acceleration in the interval t = 0 to t = 5 s, from first equation of motion, acceleration can be obtained as:
v = u + at
Where, v = Final velocity of the particle
12 = 0 + a' $\times$ 5
$a' = \frac{12}{5} = 2.4\text{m/s}^2$
Again, from first equation of motion, we have
v = u + at = 0 + 2.4 $\times$ 2 = 4.8 m/s
Distance travelled by the particle between time 2 s and 5 s i.e., in 3 s
$\text{s}_1 = \text{u}'\text{t} + \frac{1}{2}\text{a}'\text{t}^2$
$= 4.8 \times 3 + \frac{1}{2} \times 2.4 \times (3)^2$ = 25.2 m ....(ii)
For distance s_2:
Let a'' be the acceleration of the particle between time t = 5 s and t = 10 s.
From first equation of motion,
v = u + at (where v = 0 as the particle finally comes to rest)
0 = 12 + $a''\times$ 5
$a'' = \frac{-12}{5}$ = -2.4m/s^2
Distance travelled by the particle in 1s (i.e., between t = 5 s and t = 6 s)
$\text{s}_2 = \text{u}''\text{t} + \frac{1}{2}\text{at}^2$
$= 12 \times a + \frac{1}{2}(-2.4) \times (1)^2$ = 12 - 1.2 = 10.8m ....(iii)
From equations (i), (ii), and (iii), we get
s = 25.2 + 10.8 = 36 m
$\therefore$ Average speed $= \frac{36}{4} = 9\text{m/s}$

18. i. B is ahead of A by the distance OP = 100 km, when the motion starts.
ii. Speed of B = $\frac{QR}{PR} = \frac{150-100}{2-0}$ = 25 kmh^{-1}
iii. Since the two graphs intersect at point Q, so A will catch B after 2 hours and at a distance of 150 km from the origin.
iv. Speed of A = $\frac{QS}{OS} = \frac{150-0}{2-0}$ = 75 kmh^{-1}
$\therefore$ Difference in speeds = 75 - 25 = 50 kmh^{-1}

19. Let us divide the time interval of motion of an object under free fall into many equal intervals x and find out the distances traversed during successive intervals of time. Since u = 0, then $y = -\frac{1}{2}gt^2$
To calculate the position of the object after different time intervals $0, \tau, 2\tau, 3\tau, \ldots$ which are given in the second column of the table shown below.

t	y	y in Terms of y_0 = (-1/2)g τ^2	Distance Traversed in Successive Intervals	Ratio of Distance Traversed

0	0	0		
τ	$(-\frac{1}{2})$ g τ^2	y_0	y_0	1
2τ	-4($\frac{1}{2}$) g τ^2	4 y_0	3 y_0	3
3τ	-9($\frac{1}{2}$) g τ^2	9 y_0	5 y_0	5
4τ	-16($\frac{1}{2}$) g τ^2	16 y_0	7 y_0	7
5τ	-25($\frac{1}{2}$) g τ^2	25 y_0	9 y_0	9
6τ	-36($\frac{1}{2}$) g τ^2	36 y_0	11 y_0	11

The third column gives the position in the unit of y_0.

The fourth column shows the distances traversed in successive τs At last, the fifth column find the distances are in simple ratio 1 : 3 : 5 : 7. **Hence proved**