Motion in Straight Line

Lecture Notes: Acceleration

Definition of Acceleration

Acceleration is the rate at which an object changes its velocity. It is a vector quantity, which means it has both magnitude and direction. Acceleration occurs whenever an object speeds up, slows down, or changes direction.

Formula:

$$\text{Acceleration}(a) = \frac{\Delta \text{Velocity}(v)}{\Delta \text{Time}(t)}$$

Where:

- $\Delta \text{Velocity}$ = Change in velocity
- ΔTime = Change in time

Average Acceleration vs Instantaneous Acceleration

Average Acceleration:

- **Definition:** Average acceleration is the change in velocity divided by the total time taken for the change.
- **Formula:**
  $$\text{Average Acceleration}(\bar{a}) = \frac{v_f - v_i}{t_f - t_i}$$
  Where:
    - v_f = Final velocity
    - v_i = Initial velocity
    - t_f = Final time
    - t_i = Initial time
- **Example:**
  A car increases its velocity from 20 m/s to 60 m/s in 10 seconds. Calculate the average acceleration.
  $$\bar{a} = \frac{60\,\text{m/s} - 20\,\text{m/s}}{10\,\text{s}} = \frac{40\,\text{m/s}}{10\,\text{s}} = 4\,\text{m/s}^2$$

Instantaneous Acceleration:

- **Definition:** Instantaneous acceleration is the acceleration of an object at a specific moment in time.
- **Formula:**
  $$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$
- **Example:**
  If a car's velocity changes according to the function $v(t) = 5t^2$, find the instantaneous

acceleration at $t = 2$ seconds.
$a = \frac{d}{dt}(5t^2) = 10t$
At $t = 2$:
$a = 10 \times 2 = 20\,\text{m/s}^2$

Positive and Negative Acceleration

Positive Acceleration:

- **Definition:** Positive acceleration occurs when an object's velocity increases with time.
- **Example:**
  A car speeds up from 0 to 30 m/s in 5 seconds.
  $a = \frac{30\,\text{m/s} - 0\,\text{m/s}}{5\,\text{s}} = 6\,\text{m/s}^2$

Negative Acceleration (Deceleration):

- **Definition:** Negative acceleration occurs when an object's velocity decreases with time.
- **Example:**
  A car slows down from 25 m/s to 5 m/s in 4 seconds.
  $a = \frac{5\,\text{m/s} - 25\,\text{m/s}}{4\,\text{s}} = \frac{-20\,\text{m/s}}{4\,\text{s}} = -5\,\text{m/s}^2$

Uniform and Non-uniform Acceleration

Uniform Acceleration:

- **Definition:** Uniform acceleration occurs when an object's acceleration is constant over time.
- **Example:**
  A freely falling object under gravity has a uniform acceleration of $9.8\,\text{m/s}^2$ downwards.

Non-uniform Acceleration:

- **Definition:** Non-uniform acceleration occurs when an object's acceleration changes over time.
- **Example:**
  A car accelerates from 0 to 20 m/s in 5 seconds, then slows down to 10 m/s in the next 5 seconds.
  - First 5 seconds: $a_1 = \frac{20\,\text{m/s} - 0\,\text{m/s}}{5\,\text{s}} = 4\,\text{m/s}^2$
  - Next 5 seconds: $a_2 = \frac{10\,\text{m/s} - 20\,\text{m/s}}{5\,\text{s}} = -2\,\text{m/s}^2$

Calculations and Examples

Example Problem 1:
A car starts from rest and reaches a speed of 20 m/s in 4 seconds. Calculate the acceleration.
$a = \frac{20\,\text{m/s} - 0\,\text{m/s}}{4\,\text{s}} = 5\,\text{m/s}^2$

Example Problem 2:
A car decelerates from 30 m/s to 10 m/s in 5 seconds. Calculate the acceleration.
$a = \frac{10\,\text{m/s} - 30\,\text{m/s}}{5\,\text{s}} = \frac{-20\,\text{m/s}}{5\,\text{s}} = -4\,\text{m/s}^2$

Example Problem 3:
A bike's velocity is given by $v(t) = 4t + 3t^2$. Calculate the instantaneous acceleration at $t = 2$ seconds.
$a(t) = \frac{d}{dt}(4t + 3t^2) = 4 + 6t$

At $t = 2$:
$$a(2) = 4 + 6(2) = 4 + 12 = 16\,\mathrm{m/s}^2$$

Multiple Choice Questions (MCQs)

1. **What is acceleration?**
   - a) Change in speed
   - b) Change in velocity over time
   - c) Change in direction
   - d) Change in distance
2. **Which of the following best describes average acceleration?**
   - a) Acceleration at a specific instant
   - b) Total displacement divided by total time
   - c) Change in velocity divided by total time taken
   - d) Change in speed divided by total time taken
3. **What is instantaneous acceleration?**
   - a) Acceleration over a long period
   - b) Acceleration at a specific moment in time
   - c) Average acceleration
   - d) Change in displacement over time
4. **What is the acceleration of a car that increases its velocity from 0 to 20 m/s in 5 seconds?**
   - a) 4 m/s^2
   - b) 5 m/s^2
   - c) 20 m/s^2
   - d) 10 m/s^2
5. **What type of acceleration occurs when an object slows down?**
   - a) Positive acceleration
   - b) Negative acceleration
   - c) Uniform acceleration
   - d) Instantaneous acceleration
6. **What is the term for acceleration that remains constant over time?**
   - a) Non-uniform acceleration
   - b) Uniform acceleration
   - c) Instantaneous acceleration
   - d) Average acceleration
7. **If a car decelerates from 30 m/s to 10 m/s in 4 seconds, what is its acceleration?**
   - a) -5 m/s^2
   - b) -4 m/s^2
   - c) 5 m/s^2
   - d) 4 m/s^2
8. **Which of the following is an example of non-uniform acceleration?**
   - a) A car moving at constant speed
   - b) A free-falling object
   - c) A car that speeds up and then slows down
   - d) A pendulum at its equilibrium position
9. **Which of the following is a vector quantity?**
   - a) Speed
   - b) Distance
   - c) Acceleration

        ○ d) Time

10. **A car's velocity changes according to the function $v(t) = 6t^2$. What is the instantaneous acceleration at $t = 3$ seconds?**
    - ○ a) 18 m/s²
    - ○ b) 36 m/s²
    - ○ c) 54 m/s²
    - ○ d) 72 m/s²

11. **What is the average acceleration of a bike that accelerates from 5 m/s to 25 m/s in 10 seconds?**
    - ○ a) 2 m/s²
    - ○ b) 3 m/s²
    - ○ c) 4 m/s²
    - ○ d) 5 m/s²

12. **In which scenario does an object experience negative acceleration?**
    - ○ a) Speeding up
    - ○ b) Moving at constant speed
    - ○ c) Slowing down
    - ○ d) Changing direction

13. **What is the formula for average acceleration?**
    - ○ a) $a = \frac{v_f + v_i}{t}$
    - ○ b) $a = \frac{v_f - v_i}{t}$
    - ○ c) $a = \frac{d}{t}$