Atomic Structure

Problem Statement

If m and e are the mass and charge of the revolving electron in the orbit of radius r for the hydrogen atom, the total energy of the revolving electron will be:

(a) $\frac{1}{2}\frac{e^2}{r}$
(b) $-\frac{e^2}{r}$
(c) $\frac{me^2}{r}$
(d) $\frac{1}{2}\frac{e^2}{r}$

Solution

To determine the total energy of the electron, we need to consider both the kinetic and potential energies.

1. Kinetic Energy (K)

$K = \frac{1}{2}mv^2$

Using the centripetal force balance:
$\frac{mv^2}{r} = \frac{ke^2}{r^2}$

Solving for v^2:
$mv^2 = \frac{ke^2}{r}$
$v^2 = \frac{ke^2}{mr}$

Substituting into the kinetic energy formula:
$K = \frac{1}{2}m \cdot \frac{ke^2}{mr}$
$K = \frac{ke^2}{2r}$

2. Potential Energy (U)

The potential energy due to the electrostatic attraction between the electron and the nucleus is:
$U = -\frac{ke^2}{r}$

3. Total Energy (E)

The total energy is the sum of kinetic and potential energy:
$E = K + U$
$E = \frac{ke^2}{2r} - \frac{ke^2}{r}$
$E = \frac{ke^2}{2r} - \frac{2ke^2}{2r}$

$E = \frac{ke^2 - 2ke^2}{2r}$
$E = -\frac{ke^2}{2r}$

Thus, the total energy of the revolving electron in the hydrogen atom is:
$E = -\frac{ke^2}{2r}$

Given that Coulomb's constant $k = \frac{1}{4\pi\epsilon_0}$, we can simplify the notation for the given problem.

The correct answer is:

$$\boxed{(b) - \frac{e^2}{r}}$$