Motion in Straight Line

Lecture Notes: Equations of Motion for Uniform Acceleration

Introduction

The equations of motion describe the relationship between displacement, velocity, acceleration, and time for an object moving with uniform acceleration. These equations are fundamental in kinematics and are used to predict the future motion of an object given its initial conditions.

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1. First Equation of Motion

Equation:
$v = u + at$

- v: Final velocity
- u: Initial velocity
- a: Acceleration
- t: Time

Derivation:

1. **Starting with the definition of acceleration:**
   $a = \frac{v-u}{t}$
2. **Rearranging to solve for final velocity (v):**
   $v = u + at$

Application:

- To determine the final velocity of an object given its initial velocity, acceleration, and time of travel.

Example:
A car starts from rest and accelerates uniformly at $2\,\mathrm{m/s}^2$ for 5 seconds. What is its final velocity?
$v = u + at$
$v = 0 + (2\,\mathrm{m/s}^2 \times 5\,\mathrm{s})$
$v = 10\,\mathrm{m/s}$

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2. Second Equation of Motion

Equation:
$s = ut + \frac{1}{2}at^2$

- s: Displacement
- u: Initial velocity
- t: Time
- a: Acceleration

Derivation:

1. **Starting with the average velocity:**
   $\text{Average velocity} = \frac{u+v}{2}$
2. **Multiplying by time to get displacement:**
   $s = \text{Average velocity} \times t$
   $s = \left(\frac{u+v}{2}\right) \times t$
3. **Using the first equation of motion $v = u + at$ to substitute for v:**
   $s = \left(\frac{u+(u+at)}{2}\right) \times t$
   $s = \left(\frac{2u+at}{2}\right) \times t$
   $s = (u + \frac{1}{2}at) \times t$
   $s = ut + \frac{1}{2}at^2$

Application:

- To determine the displacement of an object given its initial velocity, acceleration, and time of travel.

Example:
A car starts from rest and accelerates uniformly at $2\,\mathrm{m/s}^2$ for 5 seconds. What is the displacement?
$s = ut + \frac{1}{2}at^2$
$s = 0 \times 5 + \frac{1}{2}(2\,\mathrm{m/s}^2)(5\,\mathrm{s})^2$
$s = 0 + \frac{1}{2}(2 \times 25)$
$s = 25\,\mathrm{m}$

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3. Third Equation of Motion

Equation:
$v^2 = u^2 + 2as$

- v: Final velocity
- u: Initial velocity
- a: Acceleration
- s: Displacement

Derivation:

1. **Starting with the first equation of motion:**
   $v = u + at$
2. **Solving for time t:**
   $t = \frac{v-u}{a}$
3. **Substituting t into the second equation of motion $s = ut + \frac{1}{2}at^2$:**
   $s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$

$$s = \frac{u(v-u)}{a} + \frac{1}{2}a\left(\frac{(v-u)^2}{a^2}\right)$$
$$s = \frac{uv-u^2}{a} + \frac{(v-u)^2}{2a}$$
$$s = \frac{2a(uv-u^2)+(v-u)^2}{2a^2}$$
$$s = \frac{2auv-2au^2+v^2-2uv+u^2}{2a}$$
$$2as = 2auv - 2au^2 + v^2 - 2uv + u^2$$
$$2as = v^2 - u^2$$
$$v^2 = u^2 + 2as$$

Application:

- To determine the final velocity of an object given its initial velocity, acceleration, and displacement.

Example:
A car accelerates uniformly from $10\,\mathrm{m/s}$ to $20\,\mathrm{m/s}$ over a distance of $100\,\mathrm{m}$. What is its acceleration?
$v^2 = u^2 + 2as$
$20^2 = 10^2 + 2a(100)$
$400 = 100 + 200a$
$300 = 200a$
$a = \frac{300}{200}$
$a = 1.5\,\mathrm{m/s^2}$

Summary

The three equations of motion for uniform acceleration provide a comprehensive framework to analyze and predict the motion of objects. They relate displacement, velocity, acceleration, and time, allowing us to solve a wide range of problems in kinematics. The derivations of these equations are based on basic principles of algebra and calculus, making them fundamental tools in physics.