Atomic Structure

Energy of an electron in the nth orbit (E_n)

The total energy of the electron is the sum of its potential energy and kinetic energy in its orbit.

The potential energy of the electron in the nth orbit is given by:

$$E_p = \frac{(Ze)(-e)}{4\pi\epsilon_0 r_n} = \frac{-Ze^2}{4\pi\epsilon_0 r_n} \quad ...(11)$$

The kinetic energy of the electron in the nth orbit is:

$$E_k = \frac{1}{2}mv_n^2 \quad ...(12)$$

From equation (3),

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2} = \frac{mv_n^2}{r_n}$$

$$\frac{Ze^2}{4\pi\epsilon_0 r_n} = mv_n^2 \quad ...(13)$$

Substituting equation (13) in (12),

$$E_k = \frac{1}{2}\left[\frac{Ze^2}{4\pi\epsilon_0 r_n}\right] = \frac{Ze^2}{8\pi\epsilon_0 r_n} \quad ...(14)$$

The total energy of an electron in its nth orbit is:

$$E_n = E_p + E_k = \frac{-Ze^2}{4\pi\epsilon_0 r_n} + \frac{Ze^2}{8\pi\epsilon_0 r_n}$$

$$E_n = \frac{-Ze^2}{8\pi\epsilon_0 r_n} \quad ...(15)$$

Substituting the value of r_n from equation (10) in equation (15),

$$E_n = \frac{-Z^2e^4}{8\epsilon_0^2 n^2 h^2} \quad ...(16)$$

For hydrogen atom, $Z = 1$,

$$E_n = \frac{-me^4}{8\epsilon_0^2 n^2 h^2}$$

Substituting the known values and calculating in electron-volts,

$$E_n = \frac{-13.6}{n^2}\,\text{eV} \quad ...(17) \quad \left(\because 1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}\right)$$