Atomic Structure

Derivation of Bohr's Radius

By Coulomb's law, the electrostatic force of attraction between the nucleus and the electron is given by:

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze)(e)}{r_n^2} \quad ...(1)$$

where ϵ_0 is the permittivity of free space.
Since the electron revolves in a circular orbit, it experiences a centripetal force:

$$\frac{mv_n^2}{r_n} = mr_n\omega_n^2 \quad ...(2)$$

where m is the mass of the electron.
For equilibrium, from equations (1) and (2),

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2} = \frac{mv_n^2}{r_n} \quad ...(3)$$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r_n^2} = mr_n\omega_n^2 \quad ...(4)$$

$$\omega_n^2 = \frac{Ze^2}{4\pi\epsilon_0 mr_n^3} \quad ...(5)$$

The angular momentum of an electron in the nth orbit is:

$$L = mv_nr_n = mr_n^2\omega_n \quad ...(6)$$

By Bohr's first postulate, the angular momentum of the electron is quantized:

$$L = \frac{nh}{2\pi} \quad ...(7)$$

From equations (6) and (7),

$$mr_n^2\omega_n = \frac{nh}{2\pi}$$

$$\omega_n = \frac{nh}{2\pi m r_n^2}$$

Squaring both sides,

$$\omega_n^2 = \frac{n^2h^2}{4\pi^2m^2r_n^4} \quad ...(8)$$

From equations (5) and (8),

$$\frac{Ze^2}{4\pi\epsilon_0 m r_n^3} = \frac{n^2h^2}{4\pi^2m^2r_n^4}$$

$$r_n = \frac{n^2h^2\epsilon_0}{\pi m Z e^2} \quad ...(9)$$

From equation (9), it is seen that the radius of the nth orbit is proportional to the square of the principal quantum number. Therefore, the radii of the orbits are in the ratio 1 : 4 : 9....

$$r_n = \frac{n^2h^2\epsilon_0}{\pi m e^2} \quad ...(10)$$

Substituting the known values in the above equation we get,

$$r_n = n^2 \times 0.53\,\text{Å}$$

If $n = 1$,

$$r_1 = 0.53\,\text{Å}$$

This is called the Bohr radius.