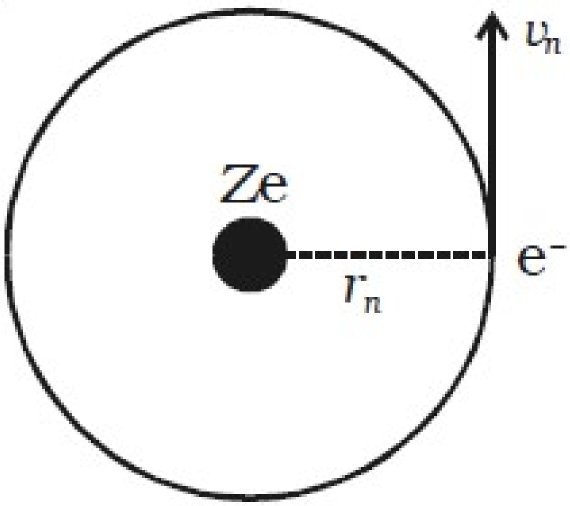
Radius of n^{th} orbit (r_n)



Z = atomic number
e = charge of proton
-e = charge of electron
r_n = radius of n^{th} orbit

By Coulomb's law, the electrostatic force of attraction between the nucleus and the electron = $\frac{1}{4\pi\varepsilon_o} \cdot \frac{(Ze)(e)}{r_n^2}$ ...(1)

where ε_o is the permittivity of the free space.

Since, the electron revolves in a circular orbit, it experiences a centripetal force, $\frac{mv_n^2}{r_n} = mr_n\omega_n^2$ ...(2)

m = mass of electron

For equilibrium, from equations (1) and (2),

$$\frac{1}{4\pi\varepsilon_o} \cdot \frac{Ze^2}{r_n^2} = \frac{mv_n^2}{r_n} \quad ...(3)$$

$$\frac{1}{4\pi\varepsilon_o} \cdot \frac{Ze^2}{r_n^2} = mr_n\omega_n^2 \quad ...(4)$$

$$\omega_n^2 = \frac{Ze^2}{4\pi\varepsilon_o m r_n^3} \qquad \ldots(5)$$

The angular momentum of an electron in n^{th} orbit is,

$$L = m v_n r_n = m r_n^2 \omega_n \qquad \ldots(6)$$

By Bohr's first postulate, the angular momentum of the electron

$$L = \frac{nh}{2\pi} \qquad \ldots(7)$$

From equations. (6) and (7),

$$m r_n^2 \omega_n = \frac{nh}{2\pi}$$

(or) $$\omega_n = \frac{nh}{2\pi m r_n^2}$$

squaring both sides,

$$\omega_n^2 = \frac{n^2 h^2}{4\pi^2 m^2 r_n^4} \qquad \ldots(8)$$

From equations (5) and (8),

$$\frac{Ze^2}{4\pi\varepsilon_o m r_n^{\ 3}} = \frac{n^2h^2}{4\pi^2 m^2 r_n^{\ 4}}$$

$$r_n = \frac{n^2h^2\varepsilon_o}{\pi m Z e^2} \qquad \ldots(9)$$

From equation (9), it is seen that the radius of the n^{th} orbit is proportional to the square of the principal quantum number. Therefore, the radii of the orbits are in the ratio 1 : 4 : 9....

$$r_n = \frac{n^2h^2\varepsilon_o}{\pi m e^2} \qquad \ldots(10)$$

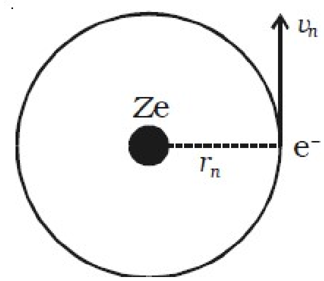
Substituting the known values in the above equation we get,

$r_n = n^2 \times 0.53$ Å

If $n = 1$, $r_1 = 0.53$Å

This is called Bohr radius.

Energy of an electron in the n^{th} orbit (E_n)



The total energy of the electron is the sum of its potential energy
and kinetic energy in its orbit

The potential energy of the electron in the n^{th} orbit is given by.

$$E_p = \frac{(Ze)(-e)}{4\pi\varepsilon_o r_n} = \frac{-Ze^2}{4\pi\varepsilon_o r_n} \qquad \ldots(11)$$

The kinetic energy of the electron in the n^{th} orbit is.

$$E_k = \frac{1}{2}m v_n^{\,2} \qquad \ldots(12)$$

$$\frac{1}{4\pi\varepsilon_o} \cdot \frac{Ze^2}{r_n^{\,2}} = \frac{m v_n^{\,2}}{r_n} \qquad \ldots(3)$$

From equation (3).

$$\frac{1}{4\pi\varepsilon_o} \cdot \frac{Ze^2}{r_n} = m v_n^{\,2} \qquad \ldots(13)$$

Substituting equation (13) in (12)

$$E_k = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_o}\ \frac{Ze^2}{r_n}\right] = \frac{Ze^2}{8\pi\varepsilon_o r_n} \qquad \ldots(14)$$

The total energy of an electron in its n^{th} orbit is,

$$E_n = E_p + E_k = \frac{-Ze^2}{4\pi\varepsilon_o r_n} + \frac{Ze^2}{8\pi\varepsilon_o r_n}$$

$$E_n = \frac{-Ze^2}{8\pi\varepsilon_o r_n} \qquad \ldots(15)$$

Substituting the value of r_n, from equation (10) in equation (15).

$$E_n = \frac{-Z^2 me^4}{8\varepsilon_o^2 n^2 h^2} \qquad \ldots(16)$$

For hydrogen atom, Z = 1

$$\therefore \quad E_n = \frac{-me^4}{8\varepsilon_o^2 n^2 h^2}$$

Substituting the known values and calculating in electron-volt.

$$E_n = \frac{-13.6}{n^2} eV \qquad \ldots(17) \qquad [\because 1eV = 1.602 \times 10^{-19} J]$$