Atomic Structure

Derivation of Bohr's Radius of Electron Orbit

Bohr's model of the hydrogen atom provides a way to calculate the radius of an electron's orbit around the nucleus. Here is the step-by-step derivation:

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1. Centripetal Force and Coulomb Force Balance

For an electron in a stable orbit around the nucleus, the centripetal force required to keep the electron in circular motion is provided by the electrostatic force of attraction between the positively charged nucleus and the negatively charged electron.

Centripetal Force:
$F_c = \frac{mv^2}{r}$

Coulomb Force:
$F_e = \frac{ke^2}{r^2}$

Here,

- m is the mass of the electron.
- v is the velocity of the electron.
- r is the radius of the electron's orbit.
- k is Coulomb's constant ($8.99 \times 10^9 \, \mathrm{Nm^2C^{-2}}$).
- e is the charge of the electron ($1.602 \times 10^{-19} \, \mathrm{C}$).

Setting the centripetal force equal to the Coulomb force:
$\frac{mv^2}{r} = \frac{ke^2}{r^2}$

Solving for v^2:
$mv^2 = \frac{ke^2}{r}$
$v^2 = \frac{ke^2}{mr}$

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2. Quantization of Angular Momentum

Bohr proposed that the angular momentum of the electron is quantized and given by:
$L = n\hbar$

Where,

- n is the principal quantum number (1, 2, 3, ...).
- $\hbar$ is the reduced Planck's constant ($\hbar = \frac{h}{2\pi}$).

The angular momentum of the electron in circular motion is:
$L = mvr$

Equating the two expressions for angular momentum:
$mvr = n\hbar$
$v = \frac{n\hbar}{mr}$

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3. Substituting for Velocity

We have two expressions for v:
$v^2 = \frac{ke^2}{mr}$
$v = \frac{n\hbar}{mr}$

Substitute v from the angular momentum quantization into the velocity squared equation:
$(\frac{n\hbar}{mr})^2 = \frac{ke^2}{mr}$
$\frac{n^2\hbar^2}{m^2r^2} = \frac{ke^2}{mr}$

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4. Solving for Radius r

Multiply both sides by r^2 and m:
$n^2\hbar^2 = ke^2r$

Solving for r:
$r = \frac{n^2\hbar^2}{ke^2m}$

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5. Bohr Radius for Hydrogen Atom

For the ground state ($n = 1$), the radius is called the Bohr radius (a_0):
$a_0 = \frac{\hbar^2}{ke^2m}$

Substitute the known values:

- $\hbar = 1.054 \times 10^{-34}\ \mathrm{Js}$
- $k = 8.99 \times 10^9\ \mathrm{Nm^2C^{-2}}$
- $e = 1.602 \times 10^{-19}\ \mathrm{C}$
- $m = 9.109 \times 10^{-31}\ \mathrm{kg}$

$a_0 = \frac{(1.054\times10^{-34}\ \mathrm{Js})^2}{(8.99\times10^9\ \mathrm{Nm^2C^{-2}})(1.602\times10^{-19}\ \mathrm{C})^2(9.109\times10^{-31}\ \mathrm{kg})}$

Calculating this, we get:
$a_0 \approx 0.529 \times 10^{-10}\ \mathrm{m}$

So, the Bohr radius a_0 is approximately $0.529\ \mathrm{\mathring{A}}$ (angstroms).

Summary

- The Bohr radius is derived using the balance of centripetal and Coulomb forces and the quantization of angular momentum.
- The derived formula for the radius of the n-th orbit is $r_n = n^2 \times 0.529\,\text{Å}$.
- For $n = 1$, the radius is known as the Bohr radius, $a_0 = 0.529\,\text{Å}$.