Solution

PERMUTATIONS AND COMBINATIONS

Class 11 - Mathematics

1. We have
   ${}^{n}C_{r}=\frac{{}^{n}P_{r}}{r!}\Rightarrow\frac{720}{r!}=120\ \ \Rightarrow r!=\frac{720}{120}=6=(3\times2\times1)=3!$
   $\Rightarrow$ r = 3
   Hence, r = 3
2. Let first arranged 5 men in the round table by 4! (by using the formula (n-1)! Mention above)
   Now there are 5 gaps created between 5 men (check the figure)
   

   So we arrange 5 ladies in this gap by 5!
   Therefore total number of ways to arrange 5, men and 5, ladies is 5!$\times$4! = 120 × 24 = 2880
3. From 4 officers and 8 jawans, 6 need to be chosen. Out of them, 1 is an officer.
   Required number of ways = ${}^{4}C_{1}\times{}^{8}C_{5}$ = $4\times\frac{8!}{5!3!}$ = $4\times\frac{8\times7\times6\times5!}{5!\times6}$ = 224
4. It is also in the form of a circle, So we need to arrange 16 flowers in Circle
   16 flowers can be arranged by 15!
   Now each flower have the same neighbours in the clockwise and anticlockwise arrangement.
   Therefore Total number of arrangement are $\frac{15!}{2}$
   = 15×14×13×12×11×10×9×8×7×6×5×4×3
5. Fixing the position of the one bead, then the remaining beads can be arranged in 7! ways.
   But, there is no distinction between the clockwise and anticlockwise arrangements.
   Therefore, the required number of arrangements = $\frac{1}{2}\times$ (7!) = $7\times6\times5\times4\times3$ = 2520
6. We have, total number of letters in MONDAY = 6
   Therefore, No. of letters to be used = 6
   $\Rightarrow$ No. of permutations = ${}^{6}_{6}\mathrm{P}=\frac{6!}{(6-6)!}=\frac{6!}{0!}$ = 720
7. 8, persons can be arranged by 7!
   Now each person have the same neighbours in the clockwise and anticlockwise arrangement
   Total number of arrangement are $\frac{7!}{2}$ = 2520
8. Given: We have 9 letters
   To Find: Number of words formed with Letter of the word ‘ALLAHABAD.’
   The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a kth kind and the rest if any, are of a different kind is = $\frac{n!}{p_1!p_2!\ldots p_k!}$
   ALLAHABAD’ consist of 9 letters out of which we have 4 A’s and 2 L’s.
   Using the above formula
   Where,
   n = 9
   p_1 = 4
   p_2 = 2
   $\frac{9!}{4!2!}$ = 7560
   7560 different words can be formed.
9. We have, $\frac{32!}{29!}$=$\frac{32\times31\times30\times(29!)}{29!}$
   Cancelling (29!) from numerator and denominator,
   = $32\times31\times30$
   = 29760

10. Let us Suppose 4 digit number as 4 boxes as,
1st, 2nd, 3rd, 4th
The 1st box can be filled with four numbers(2, 3, 4, 5) if we include 0 in the 1st box then it becomes 3 digit number(i.e. 0234 is 3 digit number, not 4 digits)
The 2nd box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.
Similarly, the 3rd box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.
Similarly, the 4th box can be filled with five numbers(0, 2, 3, 4, 5) as repetition is allowed.
Total number of ways is $4\times5\times5\times5$ = 500.
11. We have to find the possible number of ways in which we can put twenty balls in five boxes so that the first box contains only one ball when repetition of distribution of balls is allowed.
We will use the concept of multiplication because there are twenty sub-jobs dependent on each other and are performed one after the other.
The thing that is distributed is considered to have choices, not the things to which we have to give them, it means that in this problem the balls have choices more precisely four choices are there for each ball and boxes won't choose any because letters have the right to choose. But one box has the right to choose any one of the twenty balls, so the first box has twenty choices.
The number of ways in which we can put nineteen balls in four boxes where the repetition of distribution is allowed
$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{19}$
Hence the total number of ways in which we can put twenty balls in five boxes such that first box contains only one ball = 20×4^{19}
12. LHS = ${}^{2n}C_n + {}^{2n}C_{n-1}$
$= \frac{(2n)!}{n!n!} + \frac{(2n)!}{(n-1)!(2n-n+1)!}$
$= \frac{(2n)!}{n!n!} + \frac{(2n)!}{(n-1)!(n+1)!}$
$= \frac{(2n)!}{n!(n-1)!}\left[\frac{1}{n} + \frac{1}{n+1}\right]$
$= \frac{(2n)!}{n!(n-1)!}\left[\frac{2n+1}{n(n+1)}\right]$
$= \frac{(2n+1)!}{n!(n+1)!}$
RHS = $\frac{1}{2}{}^{2n+2}C_{n+1}$
$= \frac{1}{2}\left[\frac{(2n+2)!}{(n+1)!(2n+2-n-1)!}\right]$
$= \frac{1}{2}\left[\frac{(2n+2)!}{(n+1)!(n+1)!}\right]$
$= \frac{1}{2}\left[\frac{(2n+2)(2n+1)!}{(n+1)n!(n+1)!}\right]$
$= \frac{1}{2}\left[\frac{2(n+1)(2n+1)!}{(n+1)n!(n+1)!}\right]$
$= \frac{(2n+1)!}{n!(n+1)!}$
$\therefore$ LHS = RHS
13. Number of red balls = 6
Number of red balls selected = 4
Number of selections $={}^{6}C_4 = \frac{6!}{2!4!} = 15$
Number of green balls = 8
Number of green balls selected = 5
Number of selection $={}^{8}C_5 = \frac{8!}{3!5!} = 56$
The required number of selections $= 15 \times 56 = 840$
14. Given the word LATE.
Arranging the permutations of the letters of the word LATE in a dictionary:
To find: Rank of word LATE in dictionary.
First comes, words starting with letter A = 3! (3 letters, no repetation)
words starting with letter E = 3! (3 letters, no repetition)
words starting with L
words starting with LA:
LAET = 1
LATE = 1
Rank of the word LATE = 6 + 6 + 1 + 1

= 14

Hence the rank of the word LATE in arranging the letters of LATE in a dictionary among its permutations is 14

15. We have, 5. $^4P_r = 6 . (^5P_{r-1})$

$\Rightarrow 5 \cdot \frac{4!}{(4-r)!} = 6 \times \frac{5!}{[5-(r-1)]!}$

$\Rightarrow \frac{5 \cdot 4!}{(4-r)!} = \frac{6 \times 5 \times 4!}{(6-r)!}$

$\Rightarrow \quad \frac{1}{(4-r)!} = \frac{6}{(6-r)(5-r)(4-r)!}$

$\Rightarrow$ (6 - r) (5 - r) = 6

$\Rightarrow$ 30 - 11r + r^2 = 6

$\Rightarrow$ r^2 - 11r + 24 = 0

$\Rightarrow$ (r - 3) (r - 8) = 0

$\Rightarrow$ r = 3, 8

But r $\neq$ 8, because in ^4P_r, r cannot be greater than 4.

Hence, r = 3

16. Here number of questions in part I are 5 and number of questions in part II are 7. We have to select 8 questions at least 3 questions from each section. So we have required selections are 3 from part I and 5 from part II or 4 from part I and 4 from part II or 5 from part I and 3 from part II.

$\therefore$ Number of ways of selection = $^5C_3 \times {}^7C_5 + {}^5C_4 \times {}^7C_4 + {}^5C_5 \times {}^7C_3$

$= \frac{5!}{3!2!} \times \frac{7!}{5!2!} + \frac{5!}{4!1!} \times \frac{7!}{4!3!} + \frac{5!}{5!0!} \times \frac{7!}{5!0!}$

$= \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{7 \times 6 \times 5!}{5! \times 2 \times} + \frac{5 \times 4!}{4! \times 1} \times \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} + 1 \times \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!}$

$= 10 \times 21 + 5 \times 35 + 1 \times 35$

= 210 + 175 + 35 = 420

17. The number of 4-digit numbers formed by the digits 2, 3, 4 and 6

= P(4, 4) $= \frac{4!}{(4-4)!} = \frac{4!}{0!} \quad \left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$

$= \frac{4!}{1}$ [$\therefore$ 0! = 1]

= 4 $\times$ 3 $\times$ 2 $\times$ 1 = 24

   i. In this case, ‘4’ is fixed at the unit’s place. Thus, the remaining 3-digits can be 2, 3 and 6. So, the required number of 4-digits number = P (3,3)

   $= \frac{3!}{(3-3)!} \quad \left[\because {}^nP_r = \frac{n!}{(n-r)!}\right]$

   $= \frac{3!}{0!}$ = 3 $\times$ 2 $\times$ 1 = 6 ...(i)

   ii. In this case, ‘3’ is fixed at units place. Thus, remaining 3-digits can be 2, 4 and 6. So, the required number of 4-digits numbers = P (3, 3)

   P = P(3, 3) = 6 ...(ii)

   iii. The number of 4-digit numbers ending with either 3 or 4.

   P = 6 + 6 = 12 [from Eqs. (i) and (ii)]

18. i. There are 11 letters in the word 'MATHEMATICS'. Out of these letters, M occurs twice, A occurs twice, T occurs twice and the rest are all different.

   Hence, the total number of arrangements of the given letters = $\frac{11!}{(2!) \times (2!) \times (2!)}$ = 4989600.

   ii. The given word contains 4 vowels, namely A, E, A, I. Treating these 4 vowels AEAl as one letter, we have to arrange 8 letters MTHMTCS + AEAI, out of which M occurs twice, T occurs twice and the rest are all different.

   So, the number of all such arrangements = $\frac{8!}{(2!) \times (2!)}$ = 10080

   Now, out of 4 vowels, A occurs twice and the rest are all distinct.

   So, the number of arrangements of these vowels = $\frac{4!}{2!}$ = 12

   Hence, the number of arrangements in which 4 vowels are together = (10080 $\times$ 12) = 120960

19. For a number to be odd, we must have 1, 3 or 5 at the unit's place. So, there are 3 ways of filling the unit's place.

Case (i): When the repetition of digits is not allowed:

In this case, after filling the unit's place, we may fill the ten's place by any of the remaining five digits. So, there are 5 ways of filling the ten's place.

Now, the hundred's place can be filled by any of the remaining 4 digits. So, there are 4 ways of filling the hundred's place.

So, by the fundamental principle of multiplication, the required number of odd numbers = (3 $\times$ 5 $\times$ 4) = 60.

Case (ii): When the repetition of digits is allowed:

Since the repetition of digits is allowed, so after filling the unit's place, we may fill the ten's place by any of the given six digits.

So, there are 6 ways of filling the ten's place.

Similarly, the hundred's place can be filled by any of the given six digits. So, it can be filled in 6 ways.

Hence, by the fundamental principle of multiplication, the required number of odd numbers = $(3 \times 6 \times 6) = 108$