# Saitechinfo NEET-JEE Academy

## Arcsec(x): Definition and Explanation

The **arcsecant**, denoted as $\backslash\text{arcsec}(x)$ or $\sec^{-1}(x)$, is the **inverse of the secant function**. It is used to determine the angle whose secant is $x$. In other words:

$$y = \backslash\text{arcsec}(x) \quad \text{means} \quad \sec(y) = x, \text{ for } y \in \text{Range of } \backslash\text{arcsec}(x).$$

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### 1. Domain and Range of $\backslash\text{arcsec}(x)$:

**Domain:**

The secant function $\sec(x)$ is undefined at points where $\cos(x) = 0$. For $\backslash\text{arcsec}(x)$, the domain is:

$$x \in (-\infty, -1] \cup [1, \infty)$$

**Range:**

To ensure $\backslash\text{arcsec}(x)$ is a function, its range is restricted to one principal branch:

$$y \in [0, \pi], \quad y \neq \frac{\pi}{2}$$

This avoids ambiguity by excluding values where $\sec(y)$ would repeat.

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### 2. Relationship to Secant

By definition:

$$\sec(y) = x \quad \text{implies} \quad \cos(y) = \frac{1}{x}.$$

This relationship helps convert $\backslash\text{arcsec}(x)$ problems into equivalent trigonometric problems involving cosine.

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### 3. Derivative of $\backslash\text{arcsec}(x)$:

The derivative of $\backslash\text{arcsec}(x)$ is:

$$\frac{d}{dx}[\backslash\text{arcsec}(x)] = \frac{1}{|x|\sqrt{x^2 - 1}}, \quad x \in (-\infty, -1] \cup [1, \infty).$$

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### 4. Example Evaluations

1. $\backslash\text{arcsec}(2)$:

$$\sec(y) = 2 \quad \text{implies} \quad \cos(y) = \frac{1}{2}.$$

From the range of $\arcsec(x)$, $y = \frac{\pi}{3}$. So:

$$\arcsec(2) = \frac{\pi}{3}.$$

2. $\arcsec(-2)$:

$$\sec(y) = -2 \quad \text{implies} \quad \cos(y) = -\frac{1}{2}.$$

From the range of $\arcsec(x)$, $y = \frac{2\pi}{3}$. So:

$$\arcsec(-2) = \frac{2\pi}{3}.$$

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### 5. Integral Involving $\arcsec(x)$:

The integral of $\arcsec(x)$ is:

$$\int \arcsec(x)\, dx = x\arcsec(x) - \sqrt{x^2 - 1} + C$$

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### 6. Graph of $\arcsec(x)$:

- The graph of $\arcsec(x)$ is defined for $x \in (-\infty, -1] \cup [1, \infty)$.
- It is a monotonic function with a vertical asymptote at $x = -1$ and $x = 1$.

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### 7. Summary

- **Definition**: $\arcsec(x)$ is the angle whose secant is $x$, i.e., $\sec(y) = x$.
- **Domain**: $x \in (-\infty, -1] \cup [1, \infty)$.
- **Range**: $y \in [0, \pi]$, $y \neq \frac{\pi}{2}$.
- **Key Relation**: $\sec(y) = x \quad \text{implies} \quad \cos(y) = \frac{1}{x}$.

$\arcsec(x)$ is widely used in advanced calculus, engineering, and physics, particularly in problems involving inverse trigonometric functions.