

Permutations Definitions and Formulas



Permutations Overview

1. Terms:

- **Permutations:** Different ways of arranging a set of objects, where the order of arrangement matters.
- **Factorial (n!):** The product of all positive integers up to n . Used in permutation calculations.
- **Arrangement:** The specific order in which elements are arranged.

2. Symbols:

- n : The total number of objects.
- r : The number of objects to arrange.
- $P(n, r)$: The notation for the number of permutations of r objects from n distinct objects, often denoted as ${}^n P_r$.

3. Definitions:

- **Permutation Formula:** The number of ways to arrange r objects from n distinct objects is given by:

$$P(n, r) = \frac{n!}{(n - r)!}$$

This is used when order matters.

- **Factorial Function:** $n!$ is the product of all positive integers from 1 to n . For example, $4! = 4 \times 3 \times 2 \times 1 = 24$.
- **Circular Permutations:** When arranging objects in a circle, the number of permutations is given by $(n - 1)!$, because rotations of the circle produce the same arrangement.

4. Key Formulas:

- **Linear Permutations:**

$$P(n) = n!$$

This is the total number of ways to arrange n distinct objects.

- **Partial Permutations** (Selecting r objects from n):

$$P(n, r) = \frac{n!}{(n - r)!}$$

Used when only a subset of n objects are selected and arranged.

- **Circular Permutations:**

$$P_{circular}(n) = (n - 1)!$$

Used when arranging objects in a circle.

5. Theorems and Properties:

- **Multiplication Principle of Counting:** If an event can occur in m ways and another independent event can occur in n ways, then the total number of ways both events can occur is $m \times n$.
- **Derangement:** A special case of permutation where none of the objects appear in their original positions. The number of derangements of n objects is denoted as D_n .

6. Examples:

- **Example 1:** Find the number of ways to arrange 3 objects out of 5 distinct objects.

$$P(5, 3) = \frac{5!}{(5 - 3)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60$$

- **Example 2:** How many ways can 4 people sit around a circular table?

$$P_{circular}(4) = (4 - 1)! = 3! = 6$$