Saitechinfo NEET-JEE Academy

Complete Formula Set for Applications of Integrals

Integrals have numerous applications in mathematics, physics, engineering, and other fields. Below is a comprehensive formula set for key applications of integrals, categorized by context.

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1. Area Under a Curve

Vertical Slices (with respect to x):

$$\text{Area} = \int_a^b f(x)\, dx$$

Where:

- $f(x)$ is the function defining the curve,
- $[a, b]$ is the interval of integration.

Horizontal Slices (with respect to y):

$$\text{Area} = \int_c^d g(y)\, dy$$

Where:

- $g(y)$ is the function defining the curve,
- $[c, d]$ is the interval of integration.

Area Between Two Curves:

$$\text{Area} = \int_a^b \big(f(x) - g(x)\big)\, dx$$

Where:

- $f(x)$ is the upper curve,
- $g(x)$ is the lower curve.

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2. Volume of Solids of Revolution

Disk Method:

$$V = \pi \int_a^b [f(x)]^2\, dx$$

Where:

- $f(x)$ is the radius of the solid at x.

For rotation about the y-axis:

$$V = \pi \int_c^d [g(y)]^2 \, dy$$

Where $g(y)$ is the radius of the solid at y.

Washer Method:

$$V = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2 \right) dx$$

Where:

- $f(x)$ is the outer radius,
- $g(x)$ is the inner radius.

Shell Method:

$$V = 2\pi \int_a^b x \cdot f(x) \, dx$$

Where:

- x is the radius,
- $f(x)$ is the height of the shell.

For rotation about the x-axis:

$$V = 2\pi \int_c^d y \cdot g(y) \, dy$$

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3. Arc Length

Arc Length of a Curve (with respect to x):

$$L = \int_a^b \sqrt{1 + \left(f'(x)\right)^2} \, dx$$

Where $f'(x)$ is the derivative of the curve.

Arc Length of a Curve (with respect to y):

$$L = \int_c^d \sqrt{1 + \left(g'(y)\right)^2} \, dy$$

Where $g'(y)$ is the derivative of the curve.

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4. Surface Area of Solids of Revolution

Revolving About the x-Axis:

$$\text{Surface Area} = 2\pi \int_a^b f(x)\sqrt{1 + \big(f'(x)\big)^2}\, dx$$

Where $f(x)$ is the curve being revolved.

Revolving About the y-Axis:

$$\text{Surface Area} = 2\pi \int_c^d g(y)\sqrt{1 + \big(g'(y)\big)^2}\, dy$$

Where $g(y)$ is the curve being revolved.

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5. Work Done by a Variable Force

Linear Motion:

$$W = \int_a^b F(x)\, dx$$

Where $F(x)$ is the force as a function of position x.

Rotational Motion:

$$W = \int_a^b \tau(\theta)\, d\theta$$

Where $\tau(\theta)$ is the torque as a function of angular position θ.

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6. Center of Mass and Centroids

Centroid of a 2D Region (Plane Area):

For a region bounded by $y = f(x)$, $y = g(x)$, $x = a$, and $x = b$:

- x-coordinate:

$$\bar{x} = \frac{1}{A}\int_a^b x\big(f(x) - g(x)\big)\, dx$$

- y-coordinate:

$$\bar{y} = \frac{1}{A}\int_a^b \frac{\big(f(x)^2 - g(x)^2\big)}{2}\, dx$$

Where A is the total area:

$$A = \int_a^b \big(f(x) - g(x)\big)\, dx$$

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7. Probability and Statistics

Expected Value:

For a probability density function $f(x)$ over $[a, b]$:

$$E(X) = \int_a^b x f(x)\, dx$$

Variance:

$$\mathrm{Var}(X) = \int_a^b (x - E(X))^2 f(x)\, dx$$

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8. Moments of Inertia

About the x-Axis:

$$I_x = \int_a^b \big(y^2\big) \cdot \big(f(x) - g(x)\big)\, dx$$

About the y-Axis:

$$I_y = \int_a^b \big(x^2\big) \cdot \big(f(x) - g(x)\big)\, dx$$

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9. Fluid Pressure and Force

Force on a Vertical Submerged Surface:

$$F = \int_a^b \rho \cdot g \cdot h(x) \cdot w(x)\, dx$$

Where:

- ρ is the density of the fluid,
- g is gravitational acceleration,
- $h(x)$ is the depth of the surface at x,
- $w(x)$ is the width of the surface at x.

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10. Electric Charge and Current

Total Charge (on a Line, Surface, or Volume):

$$Q = \int_{\text{Region}} \sigma \, dA$$

Where σ is the charge density.

Total Current:

$$I = \int_a^b J(x) \, dx$$

Where $J(x)$ is the current density as a function of position.

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11. Average Value of a Function

$$\text{Average Value} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

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12. Length of a Parametric Curve

If the curve is given parametrically as $x = f(t)$ and $y = g(t)$, $t \in [a, b]$:

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

Key Notes:

1. Carefully set up boundaries and determine whether to integrate with respect to x or y.
2. Symmetry can often simplify calculations (e.g., doubling the integral over half the region).
3. When working with revolutions, identify whether the **disk**, **washer**, or **shell** method is most suitable.

This formula set equips you with the tools to tackle a wide range of problems involving integrals and their applications!