Heron's Formula

Example 1:

Given: A triangle with sides $a = 5$, $b = 6$, and $c = 7$.

Solution:

1. Calculate the semi-perimeter (s):

$$s = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9$$

2. Apply Heron's formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Substituting the values:

$$\text{Area} = \sqrt{9(9-5)(9-6)(9-7)}$$

$$\text{Area} = \sqrt{9 \cdot 4 \cdot 3 \cdot 2}$$

$$\text{Area} = \sqrt{216} = 14.7 \,(\text{approximately}).$$

Example 2:

Given: A triangle with sides $a = 8$, $b = 15$, and $c = 17$.

Solution:

1. Calculate the semi-perimeter (s):

$$s = \frac{a+b+c}{2} = \frac{8+15+17}{2} = 20$$

2. Apply Heron's formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Substituting the values:

$$\text{Area} = \sqrt{20(20-8)(20-15)(20-17)}$$

$$\text{Area} = \sqrt{20 \cdot 12 \cdot 5 \cdot 3}$$

$$\text{Area} = \sqrt{3600} = 60.$$

Example 3:

Given: A triangle with sides $a = 13$, $b = 14$, and $c = 15$.

Solution:

1. Calculate the semi-perimeter (s):

$$s = \frac{a + b + c}{2} = \frac{13 + 14 + 15}{2} = 21$$

2. Apply Heron's formula:

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

Substituting the values:

$$\text{Area} = \sqrt{21(21 - 13)(21 - 14)(21 - 15)}$$

$$\text{Area} = \sqrt{21 \cdot 8 \cdot 7 \cdot 6} \qquad = 84 \text{ square units}$$