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## Fundamental Theorems of Integral Calculus

The Fundamental Theorems of Calculus (FTC) are cornerstones of integral calculus, linking differentiation and integration. These theorems provide a framework for evaluating definite integrals and understanding their relationship to antiderivatives.

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### First Fundamental Theorem of Calculus (FTC-1)

If $f(x)$ is continuous on $[a, b]$ and $F(x)$ is an antiderivative of $f(x)$, then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

**Interpretation:**

- The definite integral of a function $f(x)$ over an interval $[a, b]$ equals the difference in the values of its antiderivative $F(x)$ evaluated at $b$ and $a$.
- This theorem allows computation of definite integrals using antiderivatives, bypassing the need for Riemann sums.

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### Second Fundamental Theorem of Calculus (FTC-2)

If $f(x)$ is continuous on $[a, b]$, then the function $g(x)$, defined as:

$$g(x) = \int_a^x f(t)\,dt,$$

is differentiable, and its derivative is:

$$g'(x) = f(x)$$

**Interpretation:**

- Integration and differentiation are inverse processes.
- The rate of change of the accumulation function $g(x)$ is the original function $f(x)$.

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## Applications of Fundamental Theorems

### 1. Area Under a Curve

Using FTC-1, the total area under a curve $y = f(x)$ from $x = a$ to $x = b$ can be calculated as:

$$\text{Area} = \int_a^b f(x)\,dx$$

### 2. Evaluating Definite Integrals

FTC-1 simplifies the computation of definite integrals:

1. Find an antiderivative $F(x)$ of $f(x)$.
2. Compute $F(b) - F(a)$.

### 3. Accumulated Change

FTC-2 allows the representation of the accumulation of a quantity as the integral of its rate of change. For example, the total displacement of an object given velocity $v(t)$ over time $[a, b]$ is:

$$\text{Displacement} = \int_a^b v(t)\,dt$$

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## Examples

### Example 1: Compute a Definite Integral

Evaluate $\int_1^4 (3x^2 + 2x)\,dx$.

1. Find the antiderivative of $f(x) = 3x^2 + 2x$:

$$F(x) = x^3 + x^2$$

2. Apply FTC-1:

$$\int_1^4 (3x^2 + 2x)\,dx = F(4) - F(1)$$

3. Compute:

$$F(4) = 4^3 + 4^2 = 64 + 16 = 80, \quad F(1) = 1^3 + 1^2 = 1 + 1 = 2$$

4. Result:

$$\int_1^4 (3x^2 + 2x)\,dx = 80 - 2 = 78$$

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### Example 2: Using FTC-2

Given $g(x) = \int_2^x t^2\,dt$, find $g'(x)$ and evaluate $g'(3)$.

1. By FTC-2:

$$g'(x) = f(x), \quad \text{where } f(x) = t^2 \text{ (replace } t \text{ with } x\text{)}.$$

2. Thus:

$$g'(x) = x^2$$

3. Evaluate $g'(3)$:

$$g'(3) = 3^2 = 9$$

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**Example 3: Application in Physics**

A particle moves along a line with velocity $v(t) = 5t^2 - 2t$ (in m/s). Find the displacement of the particle from $t = 0$ to $t = 3$.

1. Use the formula for displacement:

$$\text{Displacement} = \int_0^3 v(t)\,dt = \int_0^3 (5t^2 - 2t)\,dt$$

2. Find the antiderivative:

$$F(t) = \frac{5t^3}{3} - t^2$$

3. Apply FTC-1:

$$\int_0^3 (5t^2 - 2t)\,dt = F(3) - F(0)$$

4. Compute:

$$F(3) = \frac{5(3)^3}{3} - (3)^2 = 45 - 9 = 36, \quad F(0) = 0$$

5. Result:

$$\text{Displacement} = 36\,\text{m}$$

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## Key Points

- FTC-1 simplifies definite integrals using antiderivatives.
- FTC-2 connects integration with differentiation, enabling us to model accumulation functions.
- Applications span areas, displacement, total change, and solving problems in physics, engineering, and economics.