Definitions and Formulas

Combinations Overview

1. Terms:

- **Combinations**: Refers to the selection of objects from a set where the order does not matter.
- **Subset**: A part of a larger set, relevant in combinations as it refers to how many objects are chosen.

2. Symbols:

- n: The total number of objects.
- r: The number of objects to be selected.
- $C(n, r)$: The number of combinations of r objects from n distinct objects, denoted as ${}^nC_r$.

3. Definitions:

- **Combination Formula**: The number of ways to choose r objects from n distinct objects without regard to the order is given by:

  $$C(n, r) = \frac{n!}{r!(n-r)!}$$

  This is used when order does **not** matter.
- **Binomial Coefficient**: The combination formula $C(n, r)$ is also referred to as the binomial coefficient, often written as $\binom{n}{r}$.

4. Key Formulas:

- **Combination Formula**:

  $$C(n, r) = \frac{n!}{r!(n-r)!}$$

  This formula counts how many ways r objects can be selected from n distinct objects without regard to the order.
- **Complement Rule**: The combination for selecting r objects from n can also be written using the complement:

  $$C(n, r) = C(n, n-r)$$

  This means selecting r objects from n is equivalent to not selecting $n - r$ objects.
- **Sum of Combinations**: The sum of all combinations for choosing any number of objects from a set of n is:

$$\sum_{r=0}^{n} C(n,r) = 2^n$$

This is based on the fact that each object can either be included or not, leading to 2^n total possibilities.

5. Theorems and Properties:

- **Multiplication Principle of Counting**: Applies similarly to combinations, meaning the total number of ways multiple events can occur is the product of the ways each event can happen.
- **Pascal's Triangle**: The binomial coefficients $C(n,r)$ form a triangle where each entry is the sum of the two entries directly above it:

  $$C(n,r) = C(n-1,r-1) + C(n-1,r)$$

  This is a recursive property of combinations.
- **Symmetry Property**: $C(n,r) = C(n,n-r)$. This shows that choosing r objects from n is the same as leaving out $n-r$ objects.

6. Examples:

- **Example 1**: Find the number of ways to choose 3 objects from 5 distinct objects:

  $$C(5,3) = \frac{5!}{3!(5-3)!} = \frac{5 \times 4 \times 3!}{3! \times 2!} = \frac{5 \times 4}{2 \times 1} = 10$$

- **Example 2**: How many ways can a committee of 2 be chosen from a group of 6 people?

  $$C(6,2) = \frac{6!}{2!(6-2)!} = \frac{6 \times 5}{2!} = 15$$

7. Special Cases:

- **Combinations with Repetition**: If repetition of objects is allowed, the formula for combinations changes to:

  $$C(n+r-1,r) = \frac{(n+r-1)!}{r!(n-1)!}$$

  This counts the ways to select r objects from n where repetitions are allowed.
- **Combinations of Zero or All Objects**:

  $$C(n,0) = C(n,n) = 1$$

  This represents selecting no objects or all objects, which always results in 1 way.

Difference Between Permutations and Combinations

- **Permutations**: Order matters.
- **Combinations**: Order does not matter.