Solution

CIRCLES

Class 09 - Mathematics

1.

(c) 60°

Explanation:

Given, ∠DAB = 62°, ∠ABD = 58°

In △ADB, ∠DAB + ∠ABD + ∠ADB = 180°

⇒ 62° + 58° + ∠ADB = 180°

⇒ ∠ADB = 180° - 120° = 60°

Now, ∠ACB = ∠ADB = 60° (Angles in same segment are equal)

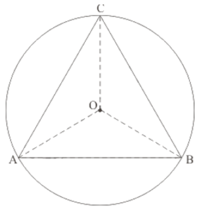
2.

(d) 120^o

Explanation:

We are given that an equilateral △ABC is inscribed in a circle with centre O. We need to find ∠BOC.

We have the following corresponding figure.



We are given AB = BC = AC

Since the sides, AB, BC, and AC are these equal chords of the circle.

Hence,

∠AOB + ∠BOC + ∠AOC = 360

==> ∠BOC + ∠BOC + ∠BOC = 360

==> 3∠BOC = 360

==> ∠BOC = $\frac{360}{3}$

==> ∠BOC = 120^o

3.

(b) 110°

Explanation:

Given, ∠BEC = 130° and ∠ECD = 20°

Now, ∠ABD = ∠ACD (Angles in the same segment)

∴ ∠ABD = 20° Now, in △AEB

∠EBA + ∠BAE = ∠BEC (exterior angle property)

⇒ 20° + ZBAC = 130° ⇒ ZBAC = 110°

4.

(d) 60°

Explanation:

Given, ∠PQR = 150°

∴ Reflex ∠POR = 2∠PQR = 2(150°) = 300°

Now, ∠POR = 360° - Reflex ∠POR = 360° - 300° = 60° ...(i)

Also, OP = OR ⇒ ∠OPR = ∠ORP ...(ii) (Angles opposite to equal sides of a triangle are equal)

In △OPR, ∠OPR + ∠ORP + ∠POR = 180°

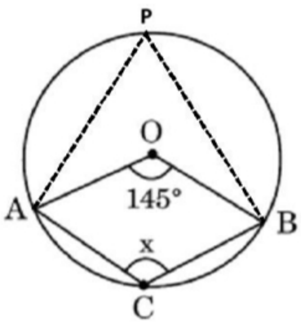
$\Rightarrow 2\angle OPR + 60° = 180°$ [From (i) & (ii)]
$\Rightarrow 2\angle OPR = 120° \Rightarrow \angle OPR = 60°$

5.

(d) 56°
Explanation:
Given, $\angle BAC = 56°$
Since, angles in same segment are equal.
$\therefore \angle BAC = \angle BDC = 56°$

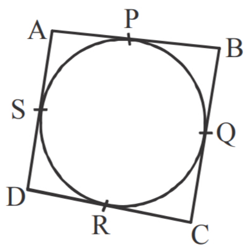
6. $\angle QRS + \angle SPQ = 180^o$ (opposite angles of cyclic quadrilateral)
$110^o + \angle SPQ = 180^o$
$\Rightarrow \angle SPQ = 180^o - 110^o = 70^o$

7. We have,
AP = 12 cm
AP = AQ = 12 cm ...(i)[$\because$ Length of tangents from an external point to a circle are equal]
From pt. B
BP = BD ...(ii)[$\because$ Length of tangents from an external point to a circle are equal]
From pt, C
CD = CQ ...(iii)[$\because$ Length of tangents from an external point to a circle are equal]
Now, Perimeter of $\triangle ABC$ = AB + BC + AC
= AB + BC + CA
$\Rightarrow$ AB + BD + CD + CA
$\Rightarrow$ AB + BP + CQ + CA .....from eqn (ii) and (iii)
$\Rightarrow$ AP + AQ [$\because$AB + BP and CQ + CA = AQ]
$\Rightarrow$ 12 + 12
= 24 cm

8. 

Take a point P on the circumference and join AP & BP.
$\angle APB = \frac{1}{2} \times 145^\circ = 72.5^\circ$
$\angle APB + \angle ACB = 180^\circ$
$\Rightarrow \angle ACB = 107.5^\circ$ or $x = 107.5^\circ$

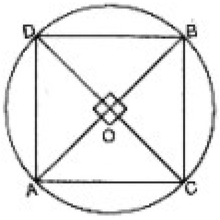
9. Let the circle touches the sides AB, BC, CD and AD at P, Q, R and S respectively.



$\therefore$ AP = AS
BP = BQ
DR = DS
CR = CQ
adding, we get (AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)
$\therefore$ AB + CD = BC + AD

10. $\angle BCD = \angle BAD = 65^\circ$ $\angle s$ of the same segment are equal.

11. Given: Two diameters AB and CD of a circle intersect each other at right angles.



To prove: The quadrilateral ACBD formed by joining their end points is a square.

Proof: A diameter essentially passes through the centre of the circle.

$\therefore$ Diameters AB and CD intersect each other at O, the centre of the circle.

$\angle A = \angle B = \angle C = \angle D = 90^\circ (\text{ each })$ [$\because$ Angle in a semi-circle is 90^o]

Quadrilateral ACBD is a rectangle .... (1)

In $\triangle OAC$ and ΔOAD

$\angle AOC = \angle AOD$ |Each = 90^o

OA = OA [common]

OC = OD [Radii of the same circle]

$\therefore \Delta OAC \cong \Delta OAD$ [SAS]

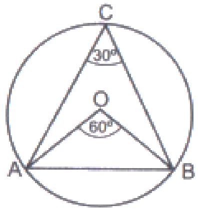
$\therefore AC = AD$ .... (2) [c.p.c.t]

In view of (1) and (2)

Quadrilateral ABCD is a square

12. Since chord of a circle is equal radius, so we have AB = OA = OB.

Therefore, ABC is an equilateral triangle.



Since each angle of an equilateral triangle is 60°, so we have $\angle AOB = 60^\circ$

Since angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle, so we have

$\angle AOB = 2\angle ACB$

Hence, $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60^\circ = 30^\circ$

13. Join PQ.

Now, quad. AQPD is cyclic

$\Rightarrow \angle QAD + \angle QPD = 180^\circ$

$\Rightarrow \quad 80^\circ + \angle QPD = 180^\circ$

$\Rightarrow \angle QPD = 180^\circ - 80^\circ = 100^\circ$

i. The side CP of quad. QBCP is produced to D.

$\therefore \ \angle QBC = \angle QPD = 100^\circ [\text{ ext. } \angle = \text{ int. opp. } \angle]$

ii. Side AQ of cyclic quad. PDAQ is produced to B.

$\therefore \ ext. \angle PQB = \angle ADP = 84^\circ$ [ext. $\angle =$ int. opp. $\angle$]

In the cyclic quad. QBCP, we have

$\angle PQB + \angle BCP = 180^\circ$

$\Rightarrow 84^\circ + \angle BCP = 180^\circ$

$\Rightarrow \angle BCP = 180^\circ - 84^\circ = 96^\circ$

14. Given : PQ and RS are two parallel chords of a circle and lines RP and SQ intersect each other at O.

To prove: OP = OQ,

Proof : PQ || RS

and transversal OR intersects them

$\therefore \angle A = \angle 3$ .... (1) |Corresponding $\angle s$

$\because$PQSR is a cyclic quadrilateral

$\therefore \angle 2 = \angle 1$ .... (2)

|∵An exterior angle of a cyclic quadrilateral is equal to its interior opposite angle

From (1) and (2),

$\angle 2 = \angle 3$

∴ OP = OQ |Sides opp. to equal angles

15. From the given figure we have

OC = OB (Radii of the same circle)

∴∠OBC = ∠OCB

⇒ ∠OBC = 57°

In △OBC, we have

∠OBC + ∠BOC + ∠OCB = 180° (by angle sum property)

57°+ ∠BOC + 57° = 180°

⇒ ∠BOC = 66°

In △OAB, we have

∠OAB + ∠OBA + ∠AOB = 180° (∵ AO = OB ∴∠OAB = ∠OBA) (by angle sum property)

30° + 30° + ∠AOB = 180°

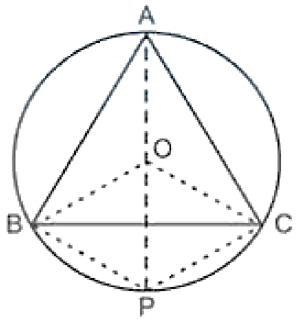
⇒ ∠AOB = 180° - 60°

⇒ ∠AOB = 120°

∠AOC = ∠AOB - ∠BOC = 120° - 66° = 54°

Thus, ∠AOC = 54° and ∠BOC = 66°

16. Since equal chords of a circle subtends equal angles at the centre, so we have chord AB = chord AC [Given]



So $\angle \mathrm{AOB} = \angle \mathrm{AOC}$ ...(i)

since the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore \quad \angle \mathrm{APC} = \frac{1}{2} \angle \mathrm{AOC}$ ...(ii)

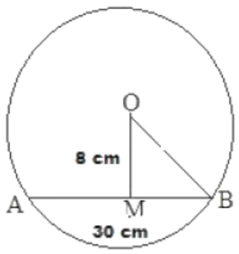
and $\angle \mathrm{APB} = \frac{1}{2} \angle \mathrm{AOB}$ ...(iii)

$\therefore \quad \angle \mathrm{APC} = \angle \mathrm{APB}$ [from (i), (ii) and (iii)]

Hence, PA is the bisector of ∠BPC.

17. Let AB be the chord of the given circle with centre O. The perpendicular distance from the centre of the circle to the chord is 8 cm.

Join OB.



Then OM = 8 cm and AB = 30 cm

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore MB = \left(\frac{AB}{2}\right) = \left(\frac{30}{2}\right) \mathrm{cm} = 15 \mathrm{cm}$

From the right angled ΔOMB, we have:

$OB^2 = OM^2 + MB^2$ {paythgoras theorem}

⇒ $OB^2 = 8^2 + 15^2$

$\Rightarrow$ $OB^2 = 64 + 225$

$\Rightarrow$ $OB^2 = 289$

$\Rightarrow OB = \sqrt{289}\text{cm}$ =17 cm

Hence, the required length of the radius is 17 cm.

18. i. $\angle QPR$

$\because$ PR is a diameter

$\therefore \angle PRQ = 90^\circ$ |Angle in a semi-circle is 90^o

In $\triangle$PQR

$\angle QPR + \angle PRQ + \angle PQR = 180^\circ$ |Angle sum property of a triangle

$\Rightarrow \angle QPR + 90^\circ + 65^\circ = 180^\circ$

$\Rightarrow \angle QPR + 155^\circ = 180^\circ$

$\Rightarrow \angle QPR = 180^\circ - 155^\circ$

$\Rightarrow \angle QPR = 25^\circ$

ii. $\angle$PRS

$\because$ PQRS is a cyclic quadrilateral

$\therefore \angle PSR + \angle PQR = 180^\circ$

|$\because$ Opposite angles of a cyclic quadrilateral are supplementary

$\Rightarrow \angle PSR + 65^\circ = 180^\circ$

$\Rightarrow \angle PSR = 180^\circ - 65^\circ$

$\Rightarrow \angle \text{PSR} = 115^\circ$

In $\triangle PSR$

$\angle PSR + \angle SPR + \angle PRS = 180^\circ$ |Angles sum property of a triangle

$\Rightarrow 115^\circ + 40^\circ + \angle PRS = 180^\circ$

$\Rightarrow 155^\circ + \angle PRS = 180^\circ$

$\Rightarrow \angle PRS = 180^\circ - 155^\circ$

$\Rightarrow \angle PRS = 25^\circ$

iii. $\angle QPM$

$\therefore$ PQ is a diameter

$\therefore \angle \text{PMQ} = 90^\circ$ |$\because$ Angle is a semi-circle is 90^o

In $\triangle PMQ$

$\angle PMQ + \angle PQM + \angle QPM = 180^\circ$ |Angle sum property of a triangle

$\Rightarrow 90^\circ + 50^\circ + \angle QPM = 180^\circ$

$\Rightarrow 140^\circ + \angle QPM = 180^\circ$

$\Rightarrow \angle QPM = 180^\circ - 140^\circ$

$\Rightarrow \angle QPM = 40^\circ$

19. i. We know that the opposite angles of a cyclic quadrilateral are supplementary.

$\therefore \angle BAD + \angle BCD = 180^\circ$

$\Rightarrow 100^\circ + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 100^\circ = 80^\circ$

ii. AB || DC and DA is the transversal.

$\therefore \angle ADC + \angle BAD = 180^\circ$ [sum of internally opposite angles]

$\Rightarrow \angle ADC + 100^\circ = 180^\circ$

$\Rightarrow \angle ADC = 180^\circ - 100^\circ = 80^\circ$

iii. Using the fact that the opposite angles of a cyclic quadrilateral are supplementary, we have

$\angle ABC + \angle ADC = 180^\circ$

$\Rightarrow \angle ABC + 80^\circ = 180^\circ$ $[\because \angle ADC = 80^\circ]$

$\Rightarrow \angle ABC = 180^\circ - 80^\circ = 100^\circ$

==> $\angle$ABC =100°

$\therefore \angle BCD = 80^\circ, \angle ADC = 80^\circ$ and $\angle ABC = 100^\circ$