Saitechinfo NEET JEE Academy

43, Haridoss Main Rd, CH-99. Ph:9444929163

RELATIONS AND FUNCTIONS

Class 12 - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 45**

1. Let A be the set of all points in a plane and let O be the origin. Let R = {(P, Q) : OP = OQ}. Then, R is **[1]**

   a) An equivalence relation
   b) Symmetric and transitive but not reflexive
   c) Reflexive and symmetric but not transitive
   d) Reflexive and transitive but not symmetric

2. Let S be the set of all real numbers and let R be a relation on S, defined by a Rb ⇔ |a - b| < 1. Then, R is **[1]**

   a) Reflexive and symmetric but not transitive
   b) Reflexive and transitive but not symmetric
   c) Symmetric and transitive but not reflexive
   d) An equivalence relation

3. Let f : R → R be defined by f(x) = $2x^3 + 2x^2 + 300x + 5 \sin x$ then f is **[1]**

   a) one-one onto
   b) one-one into
   c) many one onto
   d) many one into

4. Which of the following functions from $A = \{x : -1 \leq x \leq 1\}$ to itself are bijections? **[1]**

   a) h(x) = |x|
   b) $k(x) = x^2$
   c) $f(x) = \frac{x}{2}$
   d) $g(x) = \sin\left(\frac{\pi x}{2}\right)$

5. Let f: (-1, 1) → B where f(x) = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is one-one and onto, then B equals **[1]**

   a) $\left(0, \frac{\pi}{2}\right)$
   b) $\left[0, \frac{\pi}{2}\right]$
   c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
   d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

6. Give an example of a function which is one-one and onto. **[1]**
7. Show that the function f : R → R: f(x) = x^3 is one-one and onto **[1]**
8. Prove that the greatest integer function f: $\mathbb{R} \to \mathbb{R}$, given by f(x) = [x], is neither one-one nor onto. **[1]**
9. Show that the function f : R → R: f(x) = $1 + x^2$ is many-one into. **[1]**
10. If A = {a, b, c, d} and f = {a, b), (b, d), (c, a), (d, c)}, show that f is one-one from A onto A. Find f^{-1} **[1]**
11. Show that the relation S in set $\mathbb{R}$ of real numbers defined by **[3]**

    S = {(a, b): $a \leq b^3$, $a \in \mathbb{R}$, $b \in \mathbb{R}$} is neither reflexive, nor symmetric, nor transitive.
12. Let N be the set of all-natural numbers and let R be a relation in N, defined by R = {(a, b): a is a multiple of b}. Show that R is reflexive and transitive but not symmetric. **[3]**
13. Let L be the set of all lines in a plane and R be the relation in L defined as R = {(L_1, L_2) : L_1 is perpendicular to L_2}. Show that R is symmetric but neither reflexive nor transitive. **[3]**
14. Let R be the set of a non-zero real number. Then, show that f : R → R, given by f(x) $= \frac{1}{x}$ is one-one and onto. **[3]**
15. If A = {1, 2, 3, ... , 9} and R is the relation in A × A defined by (a, b) R (c, d), if a + d = b + c for (a, b), (c, d) in A × A. Prove that R is an equivalence relation. Also, obtain the equivalence class [(2, 5)]. **[3]**

16. Show that the function f : R $\rightarrow$ {x $\in$ R : -1 < x < 1} defined by $f(x)=\frac{x}{1+|x|}$, x $\in$ R is one-one and onto function. **[5]**

17. Given, A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following: **[5]**

   a. an injective mapping from A to B

   b. a mapping from A to B which is not injective

   c. a mapping from B to A.

18. Show that the function f : R - {3} $\rightarrow$ R - {1} given by $f(x)=\frac{x-2}{x-3}$ is a bijection. **[5]**

19. Give an example of a map **[5]**

   i. which is one-one but not onto

   ii. which is not one-one but onto

   iii. which is neither one-one nor onto.