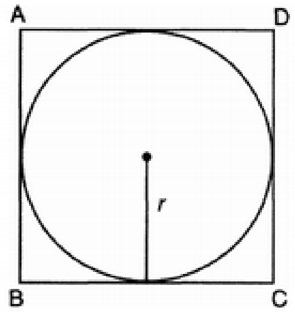
Solution

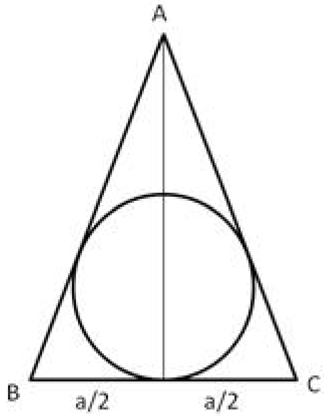
AREA RELATED TO CIRCLES

Class 10 - Mathematics

1. Side of square = 8 cm

   Side of square = diameter of circle = 8 cm
   $\therefore$ Radius of circle, r = $\frac{8}{2} = 4\text{cm}$
   Area of circle = πr^2
   = $\pi\ (4)^2$
   $= \pi \times 4 \times 4$
   $= 16\pi\ \text{cm}^2$
   So, Area of circle is $16\pi\ \text{cm}^2$.
2. When a square circumscribes a circle, the radius of the circle is half the length of the square.
   Therefore, if the radius of the circumscribed circle is a, the diameter will be 2a. It is this diameter that is equal to the length of the square.
   Therefore, the length of the square is 2a cm.
   Then area of a square =4 × length
   = 4 × 2a cm
   = 8a cm
3. Let r be the radius of the circle and angle θ subtended at the centre of the circle.
   Area of the sector of the circle $= \frac{\theta}{360} \times \pi r^2$
   Therefore, area of the sector is $= \frac{\theta}{360} \times \pi r^2$
4. Let the side of square = x units
   Diagonal of the square = $\sqrt{2}x$ units
   Diameter of the incircle = x units
   Diameter of the circumcircle = $\sqrt{2}x$ units
   $\frac{\text{Area of incircle}}{\text{Area of circumcircle}} = \frac{\pi\left(\frac{x}{2}\right)^2}{\pi\left(\frac{\sqrt{2}x}{2}\right)^2} = \frac{1}{2}$
   Ratio = 1 : 2
5. False
   Let a be the side of square.
   We are given that the circle is inscribed in the square.
   $\therefore$ Diameter of circle = Side of square = a
   $\therefore$ Radius of the circle = a/2
   Area of the circle = $\pi r^2 = \pi (a/2)^2 = (\pi a^2)/4\ cm^2$
   Hence, area of the circle is $(\pi a^2)/4\ cm^2$

6. 

Let the radius of circle be r cm

Then, $\pi r^2 = 154$

$\Rightarrow \quad r^2 = \left(154 \times \frac{7}{22}\right)$

$\Rightarrow \quad r = 7\text{cm}$

Let each side of the triangle be a cm and height be h cm

Then, $r = \frac{h}{3}$

$\Rightarrow \quad \text{h} = 3\text{r} = 21\text{cm}$

By pythagoras theorm, we have,

$h = \sqrt{a^2 - \frac{a^2}{4}}$

$= \frac{\sqrt{3\text{a}^2}}{2}$

$\Rightarrow 21 = \frac{\sqrt{3}a}{2}$

$a = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 14\sqrt{3}\text{cm}$

Therefore, Perimeter = 3a

$= (3 \times 14 \times \sqrt{3})$

$= (42 \times 1.73)\text{cm}$

= 72.66 cm

7. Radius of the sector of the circle = r cm

and angle at the centre = θ

Area of sector OAB = A cm^2

and perimeter of sector OAB = 50 cm

Area of the sector $= \pi r^2 \times \frac{\theta}{360^\circ}$

$\Rightarrow \text{A} = \pi r^2 \times \left(\frac{\theta}{360^\circ}\right)$

i. Perimeter = 2OA + arc AB

$\Rightarrow 50 = 2r + 2\pi r \times \left(\frac{\theta}{360^\circ}\right)$

$\Rightarrow 50 - 2r = 2\pi r \left(\frac{\theta}{360^\circ}\right)$

$\Rightarrow \frac{\theta}{360^\circ} = \frac{50-2r}{2\pi r} = \frac{50}{2\pi r} - \frac{2r}{2\pi r}$

$\Rightarrow \frac{\theta}{360^\circ} = \frac{25}{\pi r} - \frac{1}{\pi}$ ....(i)

$\Rightarrow \theta = 360\left(\frac{25}{\pi r} - \frac{1}{\pi}\right) = \frac{360^\circ}{\pi}\left(\frac{25}{r} - 1\right)$

ii. A = $\pi r^2 \times \left(\frac{25}{\pi r} - \frac{1}{\pi}\right) = \pi r^2 \times \frac{1}{\pi}\left(\frac{25}{r} - 1\right)$

$= r^2\left(\frac{25}{r} - 1\right) = \frac{25}{r} \times r^2 - r^2$

= 25r - r^2

Hence Proved.

8. Given, Central angle of sector = θ= 200°

and Area of the sector = 770 cm^2

We know that, area of the sector = $\frac{\pi r^2}{360^\circ} \times \theta^\circ$

$\therefore 770 = \frac{\pi r^2}{360^\circ} \times 200$

$\Rightarrow \frac{77\times18}{\pi} = r^2$

$\Rightarrow r^2 = \frac{77\times18}{22} \times 7 \Rightarrow r^2 = 9 \times 49$

$\therefore$r = 21 cm

So, radius of the sector = 21 cm.

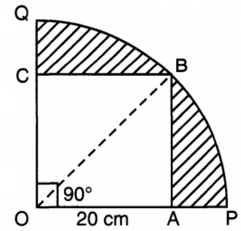
Length of the corresponding arc of this sector = $\frac{\theta}{360^o} \times 2\pi r$

$= \frac{200^o}{360^o} \times 2\pi \times 21$

$= \frac{20}{18} \times 21 \times \frac{22}{7}$

$= \frac{220}{3}\text{cm} = 73\frac{1}{3}\text{cm}$

Hence, the required length of the corresponding arc is $73\frac{1}{3}$ cm

9. 

$OB = \sqrt{OA^2 + AB^2}$

$= \sqrt{20^2 + 20^2}$

$= \sqrt{400 + 400}$

$= \sqrt{800}$

$= \sqrt{400 \times 2}$

$OB = 20\sqrt{2}$ cm or, radius = $20\sqrt{2}$

Area of shaded region = Area of sector OQBPO - Area of square OABC

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 20\sqrt{2} \cdot 20\sqrt{2} - (20)^2$

$= \frac{1}{4} \times 3 \cdot 14 \times 800 - 400$

$= 2(314) - 400$

$= 628 - 400$

$= 228$

Required Area = 228 cm^2.

10. Fencing is made on circumference ($2\pi r$) of circular field. So, we require radius for it.

Area of the circular playground = 22176 m^2

$\Rightarrow \pi r^2 = 22176$

$\Rightarrow \frac{22}{7}r^2$ = 22176

$\Rightarrow r^2 = \frac{7\times22176}{22}$

$\Rightarrow r^2 = \sqrt{7 \times 1008}$

$\Rightarrow$ r = $\sqrt{7 \times 7 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}$

$\Rightarrow$ r = 7 × 3 × 2 × 2

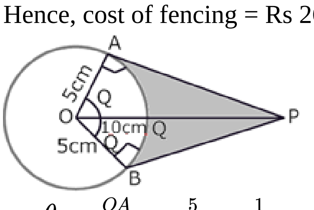
$\Rightarrow$ r = 84 m

$\therefore$ Length of fencing = Circumference of circle

= $2\pi r = 2 \times \frac{22}{7} \times 84 = 24 \times 22$ m

So, Cost of fencing = 50 × 24 × 22 = 26400

Hence, cost of fencing = Rs 26400.

11. 

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

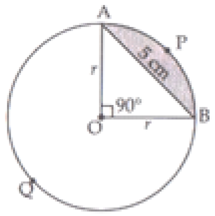
$\Rightarrow \theta = 60^o$

$\Rightarrow \angle AOB = 2 \times \theta = 120^o$

$\therefore$ ARC AB = $\frac{120\times\ 2\times\pi\times5}{360}cm = \frac{10\pi}{3}cm \left[\because l = \frac{\theta}{360} \times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley
= Circumference of the rim - length of arc AB
= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm
= $\frac{20\pi}{3}$cm
Now, the area of sector OAQB = $\frac{120\times\pi\times5\times5}{360}cm^2 = \frac{25\pi}{3}cm^2$ $\left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$
Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2
$\left[\because AP = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}\ cm\right]$
Hence, shaded area = $25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3} - \pi]\ cm^2$

12. Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.
Here, we are given that θ = 90°
Area of $\triangle$OAB = $\frac{1}{2}$Base $\times$ Altitude = $\frac{1}{2}$r $\times$ r = $\frac{1}{2}r^2$



Area of minor segment APB
$= \frac{\pi r^2\theta}{360^\circ}$ – Area of $\triangle$AOB
$= \frac{\pi r^2 90^\circ}{360^\circ} - \frac{1}{2}r^2$
$\Rightarrow$ Area of minor segment = $\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ ...(i)
Area of major segment AQB = Area of circle – Area of minor segment
= $\pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$
$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ ...(ii)
Difference between areas of major and minor segment
$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$
$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$
$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$
In right $\triangle$OAB,
$r^2 + r^2 = AB^2$
$\Rightarrow 2r^2 = 5^2$
$\Rightarrow r^2 = \frac{25}{2}$
Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right]$ cm^2

13. We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.
For the first triangle, we have a = 35, b = 53 and c = 66.
$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77\text{cm}$
Let Δ_1 be the area of the first triangle. Then,
$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$
$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77 \times 42 \times 24 \times 11}$
$\Rightarrow \quad \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11} = \sqrt{7^2 \times 11^2 \times 6^2 \times 2^2} = 7 \times 11 \times 6 \times 2 = 924\text{cm}^2$ ...(i)
For the second triangle, we have a = 33,b = 56,c = 65
$\therefore \quad s = \frac{a+b+c}{2} = \frac{33+56+65}{2} = 77\text{cm}$
Let Δ_2 be the area of the second triangle. Then,
$\Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$
$\Rightarrow \quad \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$
$\Rightarrow \quad \Delta_2 = \sqrt{77 \times 44 \times 21 \times 12} = \sqrt{7 \times 11 \times 4 \times 11 \times 3 \times 7 \times 3 \times 4} = \sqrt{7^2 \times 11^2 \times 4^2 \times 3^2}$
$\Rightarrow \quad \Delta_2 = 7 \times 11 \times 4 \times 3 = 924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2$ = 924 + 924

$\Rightarrow \quad \frac{22}{7} \times r^2 = 1848$

$\Rightarrow \quad r^2 = 1848 \times \frac{7}{22} = 3 \times 4 \times 7 \times 7 \Rightarrow r = \sqrt{3 \times 2^2 \times 7^2} = 2 \times 7 \times \sqrt{3} = 14\sqrt{3}\text{cm}$

14. **(d)** A is false but R is true.

Explanation: 2πr = 22

r = 3.5 cm

Area of the circle = $\frac{22}{7} \times 3.5 \times 3.5$ = 38.5 cm^2

15. **(c)** A is true but R is false.

Explanation: A is true but R is false.

16. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Area swept by minute hand in 5 minutes

= $\frac{\theta}{360} \times \pi r^2 = \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

= $\frac{77}{6} = 12\frac{5}{6}$ cm^2 ... (Angle in 5 minutes by minute hand is 30^o)

17. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Given,

$\frac{2\pi r_1}{2\pi r_2} = \frac{2}{3}$

$\frac{r_1}{r_2} = \frac{2}{3}$

Now, ratio of their areas be

$\frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Also, the circumference of the circle = 2πr

18. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: Area of a segment = $\frac{\theta}{360^\circ} \times \pi r^2 - \frac{1}{2}r^2 \sin\theta$

19. **(c)** 50.4 km

Explanation: In a week, Mohit drives his motorbike 3 days to go to college

$\therefore$ Total distance travelled by Mohit through motorbike = 2 $\times$ 4.2 $\times$ 6 = 50.4 km

20. **(c)** 26.4 km

Explanation: In a week Mohit rides his bicycle 3 days to go to college.

$\therefore$ Total distance travelled by Mohit through bicycle

= Length of Arc $\widehat{ACB} \times 6$

= $\frac{\theta}{360^\circ} \times 2\pi r \times$ 6 = $\frac{60^\circ}{360^\circ} \times$ 2 $\times \frac{22}{7} \times$ 4.2 $\times$ 6 = 26.4 km

21. **(a)** 9.24 km^2

Explanation: Area of sector AOB = $\frac{\theta}{360^\circ} \times \pi r^2$

= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times (4.2)^2$ = 9.24 km^2

22. **(b)** ₹ 1008

Explanation: Total cost of fuel used for a week

= ₹(20 $\times$ 50.4) = ₹ 1008

23. **(b)** 15 km

Explanation: Total length of available paths

= 4.2 + 4.2 + $\frac{\theta}{360^\circ} \times 2\pi r$

= $8.4 + \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 4.2$ = = 8.4 + 6.6 = 15 km