

DIFFERENTIAL EQUATIONS

Class 12 - Mathematics

Time Allowed: 1 hour and 30 minutes

Maximum Marks: 45

1. Verify that the given function is a solution of the corresponding diff eq. $y = \cos x + c$; $y^1 + \sin x = 0$ [1]
2. Determine order and degree (if defined) of differential equation: $y'' + (y')^2 + 2y = 0$ [1]
3. Find the product of the order and degree of the differential equation: $x \left(\frac{d^2y}{dx^2} \right)^2 + \left(\frac{dy}{dx} \right)^2 + y^2 = 0$ [1]
4. Solve the differential equation: $\frac{dy}{dx} = (e^x + 1)y$ [1]
5. Write the degree of the differential equation $\left(\frac{d^2y}{dx^2} \right)^2 + \left(\frac{dy}{dx} \right)^2 = x \sin \left(\frac{dy}{dx} \right)$ [1]
6. Find the integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ [1]
7. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ [1]
8. Write the degree of the differential equation $x \left(\frac{d^2y}{dx^2} \right)^3 + y \left(\frac{dy}{dx} \right)^4 + x^3 = 0$. [1]
9. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$. [1]
10. Write degree of the differential equation $\frac{d^2y}{dx^2} + 3 \left(\frac{dy}{dx} \right)^2 = x^2 \log \left(\frac{d^2y}{dx^2} \right)$ [1]
11. Solve $\frac{dy}{dx} + 2y = \sin x$ [3]
12. Find the particular solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$. [3]
13. Find the general solution of the differential equation: $x \frac{dy}{dx} + 2y = x^2 \log x$ [3]
14. If the interest is compounded continuously at 6% per annum, how much worth? 1000 will be after 10 years? [3]
How long will it take to double = 1000? [Given $e^{0.6} = 1.822$]
15. Solve: $\sin^{-1} \left(\frac{dy}{dx} \right) = x + y$ [3]
16. **Population** typically refers to the number of people in a single area, whether it be a city or town, region, country, or the world. Governments typically quantify the size of the resident population within their jurisdiction using a census, a process of collecting, analysing, compiling, and publishing data regarding a population. [4]



- i. The population of a village increases continuously at a rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?

ii. The population of a city increases at a rate proportional to the number of inhabitants present at any time t . If the population of the city was 200000 in 1990 and 250000 in 2000, what will be the population in 2010?

17. **Read the following text carefully and answer the questions that follow:** [4]

If the equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x , then the solution of the differential equation is given by $ye^{\int P dx} = \int Qe^{\int P dx} dx + c$, where $e^{\int P dx}$ is called the integrating factor (I.F.).

i. Find the Integrating factor of the differential equation $(1 - x^2)\frac{dy}{dx} - xy = 1$. (1)

ii. Find the solution of $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$. (1)

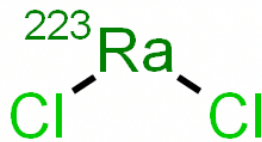
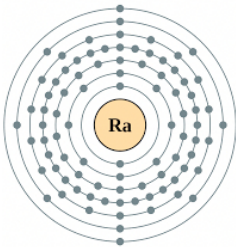
iii. Find the General solution of $\frac{dy}{dx} + y \tan x = \sec x$ (2)

OR

Find the integrating factor of differential equation $\frac{dy}{dx} - 3y = \sin 2x$. (2)

18. Radium (chemical symbol Ra) is a naturally occurring radioactive metal. Radium is a radionuclide formed by the decay of uranium and thorium in the environment. The most common isotopes of radium are Ra-226 and Ra-228. Radium-226 is part of the uranium decay series. Radium-228 and Ra-224 are part of the thorium decay series. All isotopes of radium are radioactive. Radium decays to produce radon gas. [4]

88: Radium 2,8,18,32,18,8,2



i. Radium decomposes at a rate proportional to the quantity of radium present. It is found that in 25 years, approximately 1.1% of a certain quantity of radium has decomposed. Determine approximately how long it will take for one-half of the original amount of radium to decompose? [Given $\log_e 0.989 = 0.01106$ and $\log_e 2 = 0.6931$]

ii. Experiments show that radium disintegrates at a rate proportional to the amount of radium present at the moment. Its half-life is 1590 years. What percentage will disappear in one year? [Use $e^{-\frac{\log 2}{1590}} = 0.9996$]

19. **Read the following text carefully and answer the questions that follow:** [4]

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. In other words, compound interest is the interest calculated on the principal and the interest accumulated over the previous period. It is different from simple interest, where interest is not added to the principal while calculating the interest during the next period. Let P denotes the principal at any time t and rate of interest be r % per annum.



i. Find the value of $\frac{dP}{dt}$. (1)

ii. If P_0 be the initial principal, then find the solution of differential equation formed in given situation. (1)

iii. At what interest rate will ₹ 100 double itself in 10 years? ($\log_e 2 = 0.6931$). (2)

OR

How much will ₹ 1000 be worth at 5% interest after 10 year? ($e^{0.5} = 1.648$). (2)

20. During a bacteria culture test, a **sample will be taken from your blood, urine, skin, or other parts of your body**. The type of sample depends on the location of the suspected infection. The cells in your sample will be taken to a lab and put in a special environment in a lab to encourage cell growth. [4]



The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present.

- Given the number triples in 5 hrs, find how many bacteria will be present after 10 hours.
- Also, find the time necessary for the number of bacteria to be 10 times the number of the initial presentation.

[Given $\log_e 3 = 1.0986$, $e^{2.1972} = 9$]

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