Lecture Notes: Electric Flux and Gauss's Law

1. Concept and Definition of Electric Flux

Concept of Electric Flux

- **Electric flux (Φ_E)** represents the number of electric field lines passing through a given surface.
- It gives a measure of how much **electric field is "flowing"** through a surface.
- If more field lines pass through a surface, the flux is **higher**.

Definition of Electric Flux

- **Mathematically, electric flux is given by:**

$$\Phi_E = \mathbf{E} \cdot \mathbf{A} = EA\cos\theta$$

  Where:
  - Φ_E = Electric flux (Nm²/C)
  - E = Magnitude of electric field (N/C)
  - A = Area of the surface (m²)
  - θ = Angle between the **electric field vector** and **normal to the surface**

Key Observations:

1. **If $\theta = 0^\circ$ (field perpendicular to surface):** Φ_E is **maximum** (EA).
2. **If $\theta = 90^\circ$ (field parallel to surface):** $\Phi_E = 0$ (no flux).
3. **If the surface is closed**, flux measures the net charge enclosed.

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2. Gauss's Law

Statement of Gauss's Law

- **Gauss's Law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface.**
- **Mathematical Form:**

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$$

  Where:
  - $\oint \mathbf{E} \cdot d\mathbf{A}$ = Total electric flux through a closed surface.
  - Q_{enc} = Total charge enclosed within the surface (Coulombs).
  - ε_0 = Permittivity of free space (8.85×10^{-12} C²/Nm²).

Derivation of Gauss's Law

1. Consider a **point charge Q** placed inside a **spherical Gaussian surface** of radius r.
2. The electric field at every point on the surface is:

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

3. The total flux through the sphere is:

$$\oint \mathbf{E} \cdot d\mathbf{A} = E \oint dA = E \cdot (4\pi r^2)$$

4. Substituting E:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot 4\pi r^2$$

5. Simplifying, we get:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

Thus, **Gauss's Law is derived.**

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3. Applications of Gauss's Law

A. Electric Field Due to a Uniform Spherical Charge Distribution

- Consider a **spherical charge distribution** of total charge Q.
- By symmetry, the electric field at a distance r outside the sphere is:

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

- **Key Observations:**
    1. **Inside a uniformly charged sphere ($r < R$)** → Field increases linearly with r.
    2. **Outside the sphere ($r > R$)** → The sphere behaves like a **point charge** at its center.

B. Electric Field Due to an Infinite Line Charge

- Consider an **infinite straight line** of charge with **linear charge density** λ (C/m).
- We choose a **cylindrical Gaussian surface** around the line.
- By symmetry, the electric field is **radial** and same at all points at distance r.
- Applying Gauss's Law:

$$E \cdot (2\pi r L) = \frac{\lambda L}{\varepsilon_0}$$

- **Electric Field Expression:**

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

- **Key Observation:**
    - The field **decreases as $1/r$ (not $1/r^2$ like point charge)**.

C. Electric Field Due to an Infinite Plane Sheet of Charge

- Consider a **large plane sheet** of charge with **surface charge density** σ (C/m²).
- We use a **cylindrical Gaussian surface** perpendicular to the plane.
- By symmetry, the electric field is **perpendicular to the sheet**.
- Applying Gauss's Law:

$$E \cdot (2A) = \frac{\sigma A}{\varepsilon_0}$$

- **Electric Field Expression:**

$$E = \frac{\sigma}{2\varepsilon_0}$$

- **Key Observation:**
  - The field is **constant and does not depend on distance** (unlike point or line charges).

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4. Summary of Key Formulas

Concept	Formula	Key Properties
Electric Flux	$\Phi_E = EA\cos\theta$	Flux depends on field strength and orientation.
Gauss's Law	$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$	Net flux depends only on enclosed charge.
Spherical Charge Distribution	$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$	Outside, it behaves like a point charge.
Infinite Line Charge	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$	Field decreases as $1/r$.
Infinite Plane Sheet of Charge	$E = \frac{\sigma}{2\varepsilon_0}$	Field is constant everywhere.

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5. Applications of Gauss's Law

1. **Electrostatic Shielding** – Conducting shells prevent external fields from penetrating.
2. **Parallel Plate Capacitors** – Store charge and create uniform fields.
3. **Lightning Rods** – Work due to strong field concentration at sharp points.
4. **Charged Conductors** – Charge distributes evenly on spherical conductors.

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