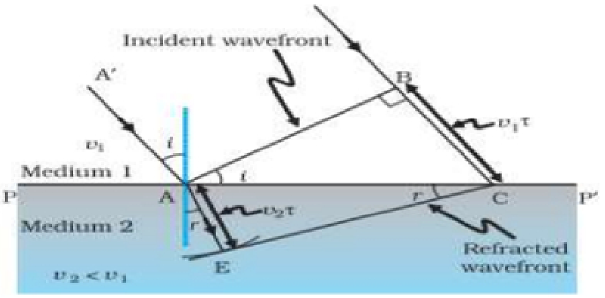
Solution

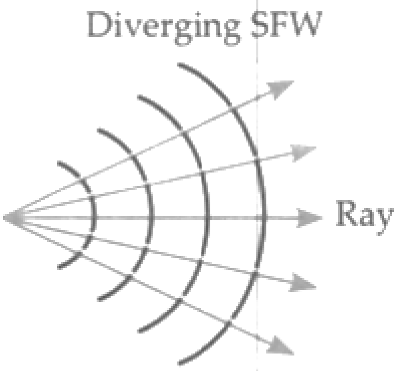
WAVE OPTICS

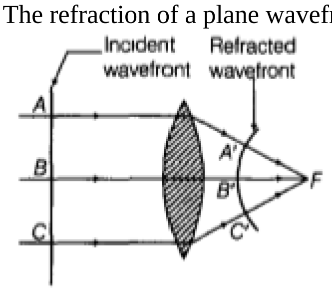
Class 12 - Physics

1. If one of the slit transmits only half of the light intensity, then the intensity in the interference pattern will vary from maximum to minimum but not become zero. Therefore contrast will be reduced & the interference pattern will become less sharp. So we can say that the intensity of maxima decreases and the intensity of minima increases.
2. 

   AB is incident wave front, incident at an angle i. Let τ be time taken by the wave front to travel distance BC.
   BC = $v_1 \tau$ where v_1 is speed of wave in medium 1.

   To determine shape of refracted wave front, we draw a sphere of radius $v_2 \tau$, where v_2 is speed of wave in medium 2.

   CE represents a tangent drawn from point C on sphere, CE is the refracted wave front.
   sin i = $\frac{BC}{AC} = \frac{v_1 \tau}{AC}$
   sin r = $\frac{AE}{AC} = \frac{v_2 \tau}{AC}$
   $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = n_{21}$
3. We want $a\theta = \lambda, \theta = \frac{\lambda}{a}$
   $10\frac{\lambda}{d} = 2\frac{\lambda}{a} \rightarrow \frac{d}{5} = \frac{1}{5}$ = 0.2 mm
4. i. for constructive interference path difference, Δp = nλ
   ii. for destructive interference path difference, Δp = $(2n+1)\frac{\lambda}{2}$, n = 0, 1, 2, 3 - -
5. Interference of light
6. Circular
7. 

8. The refraction of a plane wavefront incident on a convex lens is shown in the figure below :
   

   Refracted spherical wavefront emerges out which is converging towards the focus of a lens.
9. Linear width, $\beta_0 = 2\frac{D\lambda}{a} = \frac{2\times3\times600\times10^{-9}}{3\times10^{-3}}$

   = 1.2×10^{-3}m = 1.2 mm
10. When the film is thick (t $\simeq 20\lambda$), the path difference 2μt cos r will be so large that the conditions of both maxima and minima for different wavelength of white light will be essentially satisfied at the same value of thickness t. Difference colors will overlap each

other at all places of the film; producing general illumination of the film. No separate colours or fringes will be seen.

11. a. fringe width $\beta = \frac{D\lambda}{d}$

so as d increases β decrease so fringes will not be resolved and observable. (they do not appear separate)

b. Conditions:-

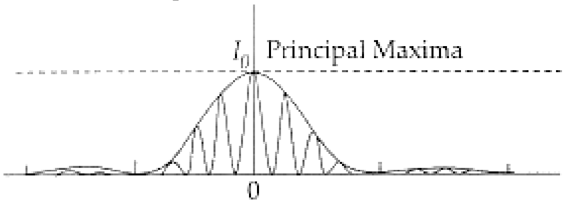
- Two sources produce waves of same frequency.
- Two sources produce waves of constant phase difference.

c. $\beta = \frac{D\lambda}{d}$

so if λ changes by 1.5λ

β, fringe width increases, so more brighter will be the fringes.

12. The diagram, given here, shows several fringes, due to double-slit interference, 'contained' in a broad diffraction peak. When the separation between the slits is large compared to their width, the diffraction pattern becomes very flat and we observe the two-slit interference pattern.



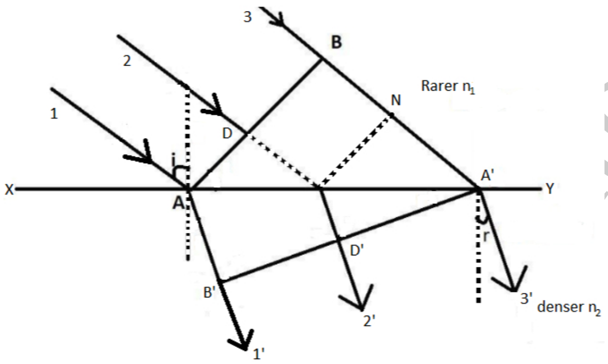
Basic features that distinguish the interference pattern from those seen in a coherently illuminated single slit.:

i. The interference pattern has a number of equally spaced bright and dark bands while the diffraction pattern has a central bright maxima which is twice as wide as the other maxima.

ii. Interference pattern is the superposition of two waves originating from two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.

iii. For a single slit of width 'a' the first null of diffraction pattern occurs at an angle of $\frac{\lambda}{a}$. At the same angle of $\frac{\lambda}{a}$, we get a maximum for two narrow slits separated by a distance a.

13. Let XY be interface and c_1 and c_2 are velocity of light in rarer and denser medium respectively.



then, $\mu = \frac{c_1}{c_2}$

μ is refractive index of medium 2 with respect to medium 1.

According to Huygens principle, every point of plane wavefront AB acts as a source of secondary wavelet. Let the secondary wavelets from B strikes XY at A' in t seconds

BA' = $c_1 \times t$

Similarly from A secondary wavelet travels in denser medium with velocity c_2

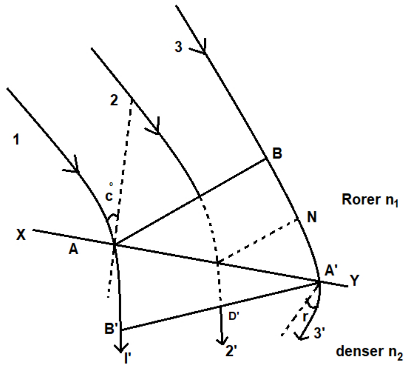
AB' = $c_2 \times t$

In $\triangle$AA'B

sin i = $\frac{BA'}{AA'} = \frac{C_1 t}{AA'}$

In $\triangle$AA'B' sin r = $\frac{AB'}{AA'} = \frac{C_2 \times t}{AA'}$

$\frac{\sin i}{\sin r} = \frac{c_1}{c_2} = u = \frac{n_2}{n_1}$ This proves snell's law of refraction.

14. 

Let XY be interface of two mediums and c_1 and c_2 are velocity of light is rarer and denser medium respectively.

Then, refractive index of denser medium relative to rarer medium $\mu=\frac{c_1}{c_2}$

Also, according to Huygens principle, every point of plane wavefront AB acts as a source of secondary wavelet. Let the secondary wavelets takes time t from B to strike at XY surface at A'

BA' = $c_1 \times$ t

in same time t wavelet from A travels with speed c_2 and strikes B'

AB' = $c_2 \times$ t

In $\triangle$AA'B

sin i = $\frac{BA'}{AA'}=\frac{C_1 t}{AA'}$

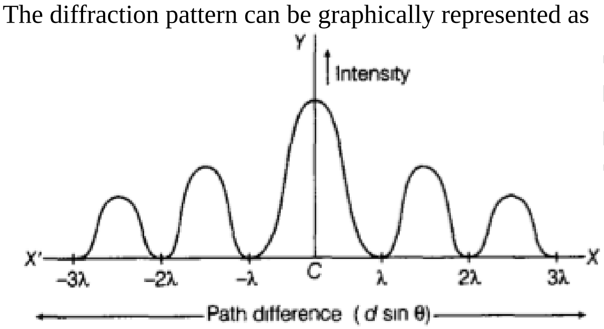
In $\triangle$AA'B'

sin r = $\frac{AB'}{AA'}=\frac{C_2 \times t}{AA'}$

$\frac{\sin i}{\sin r}=\frac{c_1}{c_2}=\mu=\frac{n_2}{n_1}$ This verifies the snell's law of refraction.

15. In case of single slit, the diffraction pattern obtained on the screen consists of a central bright band having alternate dark and weak bright band of decreasing intensity on both sides.

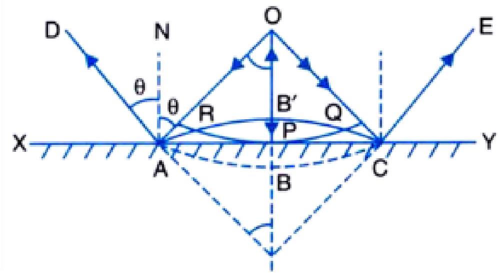
The diffraction pattern can be graphically represented as



Points to compare the intensity distribution between interference and diffraction are:

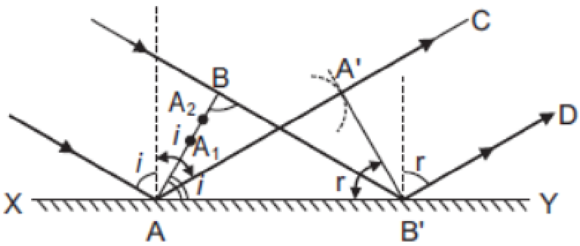
a. In the interference,it is produced due to two different wave fronts, but in diffraction, it is produced due to different parts of same wave fronts.
b. In the interference, fringe width is same size, but in diffraction, central fringe is twice as wide as other fringes.
c. In the interference, all bright fringes have same intensity, but in diffraction, all the bright fringes are not of the same intensity.
d. In interference, the widths of all the fringes are same but in diffraction, fringes are of different widths. The point C corresponds to the position of central maxima and the position$-3\lambda, 2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda.....$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima.

16. We are given a plane mirror XY and let, O be a point object at a distance OP, in front of the plane mirror. A part RPQ of the wavefront touches the plane mirror at P and from this point, spherical wavefronts start emanating. Whereas disturbance from R and Q continues moving forward, along with the normal rays OR and OQ, that reflects back v. When, disturbances from R, P, and Q reach the mirror at A, B' and C respectively, reflected spherical wavefront is formed.



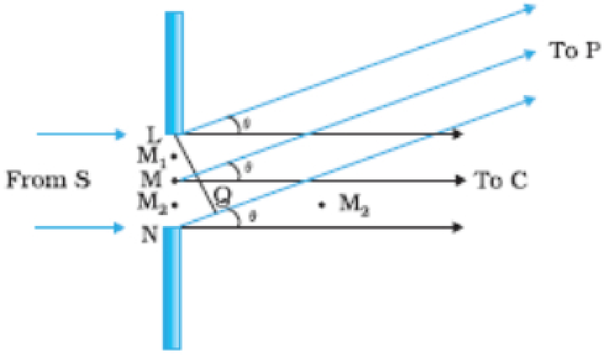
The reflected wavefront AB'C appears to start from I. Hence, I become a virtual image for 0 as a real point object. Draw AN normal to XY, hence parallel to OP.
Now, OA is the incident ray (being normal to incident wavefront ABC) and AD is the reflected ray (being normal to reflected wavefront AB'C).
Thus, $\angle \mathrm{OAN} = \angle \mathrm{DAN} = \theta$ [i = r]
But, $\angle OAN$ = alternate $\angle$AOP
and $\angle$DAN = corresponding $\angle$AlP
$\therefore \quad \angle \mathrm{AOP} = \angle \mathrm{AlP}$
$\angle \mathrm{AIP} = \angle \mathrm{AOP}$ (each θ)
$\angle AP| = \angle APO = 90^\circ$ (each 90^o)
AP is common to both
Δ_s become congruent
Hence, PI = PQ
i.e., a normal distance of the image from the mirror = normal distance of the object from the mirror.
Thus, a virtual image is formed as much behind the mirror as the object in front of it.

17. a. **Law of Reflection:** Let XY be a reflecting surface at which a wavefront is being incident obliquely. Let v be the speed of the wavefront and at time t = 0, the wavefront touches the surface XY at A. After time t, the point B of wavefront reaches the point B' of the surface. According to Huygens principle, each point of the wavefront acts as a source of secondary waves. When point A of the wavefront strikes the reflecting surface, then due to the presence of the reflecting surface, it cannot advance further; but the secondary wavelet originating from point A begins to spread in all directions in the first medium with speed v. As the wavefront AB advances further, its points A_1, A_2, A_3 K etc. strike the reflecting surface successively and send spherical secondary wavelets in the first medium.



First of all the secondary wavelet starts from point A and traverses distance AA' (=vt) in the first medium in time t. In the same time t, the point B of wavefront, after travelling a distance BB', reaches point B' (of the surface), from where the secondary wavelet now starts. Now taking A as centre we draw a spherical arc of radius AA' (= vt) and draw tangent A' B' on this arc from point B'. As the incident wavefront AB advances, the secondary wavelets start from points between A and B¢, one after the other and will touch A' B' simultaneously. According to Huygens principle wavefront A' B' represents the new position of AB, i.e., A' B' is the reflected wavefront corresponding to incident wavefront AB. Now in right-angled triangles ABB' and AA' B'
$\angle$ABB' = $\angle$ AA'B' (both are equal to 90^o)
side BB' = side AA' (both are equal to vt) and side AB' is common i.e., both triangles are congruent.
$\therefore$ $\angle$BAB' = $\angle$AB'A
i.e., incident wavefront AB and reflected wavefront A' B' make equal angles with the reflecting surface XY. As the rays are always normal to the wavefront, therefore the incident and the reflected rays make equal angles with the normal drawn on the surface XY, i.e., angle of incidence i = angle of reflection r
This is the second law of reflection. Since AB, A' B' and XY are all in the plane of paper, therefore the perpendiculars dropped on them will also be in the same plane. Therefore we conclude that the incident ray, reflected ray and the normal at the point of incidence, all lie in the same plane. This is the first law of reflection. Thus Huygens principle explains both the laws of reflection.

b. i. If the radiation of a certain frequency interacts with the atoms/molecules of the matter, they start to vibrate with the same frequency under forced oscillations. Thus, the frequency of the scattered light (Under reflection and refraction) equals to the frequency of incident radiation.
   ii. No, the energy carried by the wave depends on the amplitude of the wave, but not on the speed of the wave.

18. a. 

From diagram path difference between the waves from L and N = a sinθ

When first minimum is obtained at P then path difference $= \lambda$

[imagine the slit be divided into two halves, for each wavelets from first half of the slit has a corresponding wavelet from second half of the slit differing by a path of $\frac{\lambda}{2}$ and cancel each other]

Condition for first minimum

$\therefore \lambda = \mathrm{a} \sin \theta$

b. $\beta_{\mathrm{cm}} = \frac{2\lambda \mathrm{D}}{\mathrm{d}}$

   i. As we know that wave length (λ) of red light is more than yellow light.

   $\therefore \lambda_{\text{red}} > \lambda_{\text{yellow}}$

   So, $\therefore \beta_{\text{red}} > \beta_{\text{yellow}}$

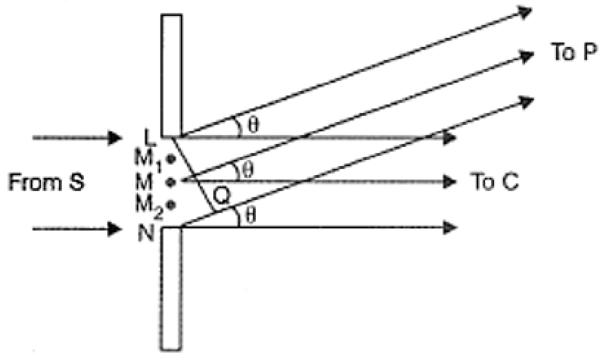
   Hence, the linear width of the central maximum will increase if monochromatic yellow light is replaced with the red light.

   ii. If the distance between the slit and screen (d) is increased then also the linear width of the central maximum will increase.

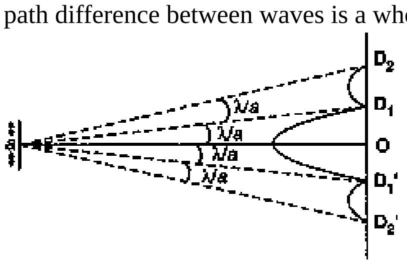
c. $10\frac{\lambda}{\mathrm{d}} = 2\frac{\lambda}{a}$

$\mathrm{a} = \frac{\mathrm{d}}{5}$ = 0.2 mm

19. i. We can regard the total contributions of the wavefront LN at some point P on the screen, as the resultant effect of the superposition of its wavelets like LM, MM_2, M_2N. These have to be superposed taking into account their proper phase differences.



We, therefore, get maxima and minima, i.e., a diffraction pattern, on the screen. **Maxima and minima** are produced when the path difference between waves is a whole number of wavelengths or an odd number of half wavelengths respectively.

ii. 

Conditions for first minima on the screen

asin θ = λ

$\Rightarrow \quad \theta = \frac{\lambda}{a}$

$\therefore$ Angular width of the central fringe on the screen (from the figure)
$= 2\theta = \frac{2\lambda}{a}$
Angular width of first diffraction fringe (From fig)
$= \frac{\lambda}{a}$
For the first diffraction, the angular width of the fringe is half that of the central fringe.

iii. Maxima becomes weaker and weaker with increasing n. This is because the effective part of the wavefront, contributing to the maxima becomes smaller and smaller, with increasing n.