Solution

INVERSE TRIGONOMETRY

Class 12 - Mathematics

1.

(b) π

Explanation:

We have,

$\tan^{-1}(\sqrt{3}) + \cos^{-1}\left(-\frac{1}{2}\right) = \tan^{-1}(\tan \frac{\pi}{3}) + \cos^{-1}(-\cos \frac{\pi}{3})$

$= \frac{\pi}{3} + \cos^{-1} |\cos(\pi - \frac{\pi}{3})| = \frac{\pi}{3} + \cos^{-1} (\cos \frac{2\pi}{3})$

$= \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi$

2.

(b) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Explanation:

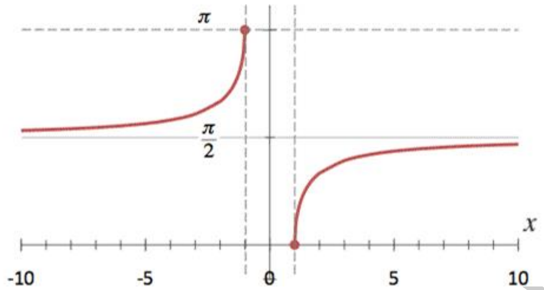
To Find: The range of $\sec^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sec^{-1}(x)$ can be obtained from the graph of

$Y = \sec x$ by interchanging x and y axes.i.e, if (a, b) is a point on $Y = \sec x$ then (b, a) is the point on the function $y = \sec^{-1}(x)$

Below is the Graph of the range of $\sec^{-1}(x)$



From the graph, it is clear that the range of $\sec^{-1}(x)$ is restricted to interval

$[0, \pi] - \left\{\frac{\pi}{2}\right\}$

3.

(b) 0.96

Explanation:

Let $\sin^{-1} (0.8) = \theta \Rightarrow \sin \theta = 0.8$

$\Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta} \Rightarrow \cos \theta = \sqrt{1 - (0.8)^2}$

$\Rightarrow \cos \theta = \sqrt{1 - 0.64} \Rightarrow \cos \theta = \sqrt{0.36}$

$\Rightarrow \cos \theta = 0.6$

$\therefore \sin(2 \sin^{-1} (0.8)) = \sin 2\theta = 2 \sin \theta \cos \theta = 2 \times 0.8 \times 0.6 = 0.96$

4.

(b) $\frac{-\pi}{10}$

Explanation:

$\sin^{-1}\left(\cos \frac{3\pi}{5}\right)$

$= \sin^{-1} \sin \left(\frac{\pi}{2} - \frac{3\pi}{5}\right)$

$= \sin^{-1} \sin \left(\frac{5\pi - 6\pi}{10}\right)$

= $\sin^{-1} \sin \left(\frac{-\pi}{10}\right)$

= $\frac{-\pi}{10}$

5.

(b) None of these

Explanation:

Domain of $\sin^{-1}x$ is [-1, 1] and domain of $\frac{1}{x}$ is IR - $\{\theta\}$

$\therefore$ Domain of f(x) = $\frac{\sin^{-1} x}{x}$ is [-1, 1] - {0}

6. **(a)** x $\in$ (2 - $\sqrt{9-2\pi}$, 2 + $\sqrt{9-2\pi}$)

Explanation:

$\because$ 1 rad = 57.75^o $\Rightarrow$ 5 rad = 288.75^o

$\Rightarrow \frac{3\pi}{2} < 5 < \frac{5\pi}{2} \Rightarrow \sin^{-1}(\sin 5) = 5 - 2\pi$

Now, $\because \sin^{-1}(\sin 5) > x^2 - 4x$

$\Rightarrow 5 - 2\pi > x^2 - 4x \Rightarrow x^2 - 4x + (2\pi - 5) < 0$

$\Rightarrow 2 - \sqrt{9-2\pi} < x < 2 + \sqrt{9-2\pi}$ ($\because x^2 - 4x + (2\pi - 5) = 0 \Rightarrow x = 2 \pm \sqrt{9-2\pi}$)

$\Rightarrow$ x $\in$ (2 - $\sqrt{9-2\pi}$, 2 + $\sqrt{9-2\pi}$)

7.

(c) [0, 1]

Explanation:

We have f(x) = $\cos^{-1}(2x - 1)$

Since, $-1 \leq 2x - 1 \leq 1$

$\Rightarrow 0 \leq 2x \leq 2$

$\Rightarrow 0 \leq x \leq 1$

$\therefore x \in [0,1]$

8. **(a)** 1

Explanation:

If x = $\frac{\sqrt{3}}{2}$, then

$\tan\left(\frac{\sin^{-1} x+\cos^{-1} x}{2}\right) = \tan\left[\frac{\sin^{-1} \frac{\sqrt{3}}{2}+\cos^{-1} \frac{\sqrt{3}}{2}}{2}\right]$

$= \tan\left(\frac{\frac{\pi}{3}+\frac{\pi}{6}}{2}\right) = \tan\left(\frac{\frac{\pi}{2}}{2}\right) = \tan\frac{\pi}{4} = 1$

9.

(d) $\frac{\pi}{6}$

Explanation:

Let the principle value be given by x

Now, let x = $\text{cosec}^{-1}(2)$

$\Rightarrow$ cosec x = 2

$\Rightarrow \text{cosec } x = \text{cosec}\left(\frac{\pi}{6}\right) \left(\because \cos\left(\frac{\pi}{6}\right) = 2\right)$

$\Rightarrow x = \frac{\pi}{6}$

10. **(a)** $\frac{-\pi}{6}$

Explanation:

Let the principle value be given by x

also, let x = $\sin^{-1}\left(\frac{-1}{2}\right)$

$\Rightarrow \sin x = \frac{-1}{2}$

$\Rightarrow$ sin x = -sin $\left(\frac{\pi}{6}\right)\left(\because \sin\left(\frac{\pi}{6}\right)=\frac{1}{2}\right)$

$\Rightarrow$ sin x = sin $\left(-\frac{\pi}{6}\right)(\because -\sin(\theta)=\sin(-\theta))$

$\Rightarrow$ x = $-\frac{\pi}{4}$

11.

(d) $\frac{2\pi}{3}$

Explanation:

$\sec^{-1}$ (sec $\frac{4\pi}{3}$) = $\sec^{-1}$ (sec$\left(\pi+\frac{\pi}{3}\right)$)

= $\sec^{-1}$ (-sec $\frac{\pi}{3}$) = $\sec^{-1}$ (-2) = π - $\sec^{-1}$ 2

$=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

12. **(a)** $\frac{-\pi}{4}$

Explanation:

Let the principle value be given by x

also, let x = $\operatorname{cosec}^{-1}(-\sqrt{2})$

$\Rightarrow$ cosec x = $-\sqrt{2}$

$\Rightarrow$ cosec x =-cosec $\left(\frac{\pi}{4}\right)\left(\because\ cosec\left(\frac{\pi}{4}\right)=\sqrt{2}\right)$

$\Rightarrow$ cosec x = cosec $\left(-\frac{\pi}{4}\right)$ ($\because$ - cosec (θ) = cosec (-θ))

$\Rightarrow$ x = $-\frac{\pi}{4}$

13.

(c) $\frac{1}{\sqrt{1+x^2}}$

Explanation:

$\cot^{-1}x=\theta \Rightarrow x=\cot\theta \Rightarrow \cot\theta=\frac{x}{1}$

$sin(cot^{-1}x)=\sin\theta=\frac{Perp.}{Hyp.}=\frac{1}{\sqrt{x^2+1}}.$

14. **(a)** $\left[-\frac{1}{2},\frac{1}{2}\right]$

Explanation:

$f(x)=\sin^{-1}2x$

Let $\sin^{-1}2x=\theta$

$\sin\theta=2x$

$-1\leq\sin\theta\leq 1$

$-1\leq 2x\leq 1$

$-\frac{1}{2}\leq x\leq\frac{1}{2}$

$x\in\left[-\frac{1}{2},\frac{1}{2}\right]$

15. $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)=?$

$\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$

$=\tan^{-1}\sqrt{3}-\left(\pi-\cot^{-1}\sqrt{3}\right)\ \left[\because \cot^{-1}(-x)=\pi-\cot^{-1}x\right]$

$=\tan^{-1}\sqrt{3}-\pi+\cot^{-1}\sqrt{3}$

$=\left(\tan^{-1}\sqrt{3}+\cot^{-1}\sqrt{3}\right)-\pi\ \left[\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$

$=\frac{\pi}{2}-\frac{\pi}{1}=\frac{-\pi}{2}$

16. According to the question, $\tan^{-1}(\sqrt{3})-\cot^{-1}(-\sqrt{3})$

$=\tan^{-1}(\sqrt{3})-\left\{\pi-\cot^{-1}(\sqrt{3})\right\}$

$\left[\because \cot^{-1}(-x)=\pi-\cot^{-1}x; x\in R\right\}$

$=\tan^{-1}\sqrt{3}-\pi+\cot^{-1}\sqrt{3}$

$=\left(\tan^{-1}\sqrt{3}+\cot^{-1}\sqrt{3}\right)-\pi$

$=\frac{\pi}{2}-\pi=-\frac{\pi}{2}\left[\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}; x\in R\right]$

is the required principal value.

17. We have, $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$= \tan^{-1}\left[2\sin\left\{2\cos^{-1}\left(\cos\frac{\pi}{6}\right)\right\}\right]$ $\left[\because \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right]$

$= \tan^{-1}\left[2\sin\left\{2\times\frac{\pi}{6}\right\}\right]$

$\left[\because \cos^{-1}(\cos\theta) = \theta; \forall\theta\in[0,\pi]\right]$

$= \tan^{-1}\left(2\sin\frac{\pi}{3}\right) = \tan^{-1}\left(2\cdot\frac{\sqrt{3}}{2}\right)$ $\left[\because \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}\right]$

$= \tan^{-1}(\sqrt{3}) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$

$\left[\because \tan\frac{\pi}{3} = \sqrt{3} \text{ and } \tan^{-1}(\tan\theta) = \theta, \forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$

18. According to the question, $\tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$

$= \tan^{-1}\left[-\sin\left(\frac{\pi}{2}\right)\right]$ $\left[\begin{array}{ll}\because \sin^{-1}(-x) & = -\sin^{-1}x \\ x\in(-1,1) & \end{array}\right]$

$= \tan^{-1}(-1)$ $\left[\because \sin\left(\frac{\pi}{2}\right) = 1\right]^{3}$

$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$ $\left[\because \tan\frac{\pi}{4} = 1\right]$

$= \tan^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$

$\left[\because \tan^{-1}(\tan\theta) = \theta; \forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$

is the required principal value.

19. $\tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$

$= \tan^{-1}\left(-\sin\left(\frac{\pi}{2}\right)\right)$

$= \tan^{-1}(-1) = -\frac{\pi}{4}$

20. We know that the value of $\sec^{-1}\left(\frac{-1}{2}\right)$ is undefined therefore,

it is outside the range i.e. R - (-1, 1).

21. Let $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) =$ x and $\cos^{-1}\left(\frac{-1}{2}\right)$ = y

$\Rightarrow$ sin x = $\left(\frac{-\sqrt{3}}{2}\right)$ and cos y = $\frac{-1}{2}$

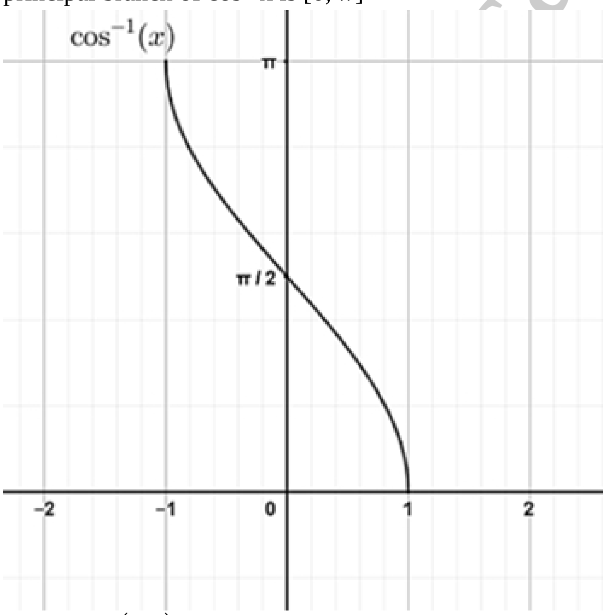
we know that the range of the principal value branch of $\sin^{-1}\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ and $\cos^{-1}$ is $(0,\pi)$

We also know that sin $\frac{-\pi}{3}$ = $\left(\frac{-\sqrt{3}}{2}\right)$ and cos $\left(\frac{2\pi}{3}\right)$ = $\frac{-1}{2}$

$\therefore$ Value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ + $\cos^{-1}\left(\frac{-1}{2}\right) = \frac{-\pi}{3}+\frac{2\pi}{3} = \frac{\pi}{3}$

22. f(x) = $\cos^{-1}$ x.

principal branch of $\cos^{-1}$x is $[0, \pi]$



23. Let $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \theta$

$\cot\theta = \frac{-1}{\sqrt{3}}$

We know that $\theta\in(0,\ \pi)$

$\cot\theta = \cot\left(\pi - \frac{\pi}{3}\right)$

$\theta=\frac{2\pi}{3}$

Therefore principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=\frac{2\pi}{3}$

24. We know that for any x $\in$ R, $\tan^{-1}$x represents an angle in $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ whose tangent is x. Therefore,

$\tan^{-1}(-\sqrt{3})=\left(\text{An angle } \theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\text{ such that } \tan\theta=-\sqrt{3}\right)$

$=-\frac{\pi}{3}$

25. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

26.

(c) A is true but R is false.

Explanation:

Explanation: Let $\sin^{-1}$ 5x = θ

$\therefore$ sin $\left(\sin^{-1}5x\right)$ = sin θ

$\Rightarrow$ 5x = sinθ

Since,-1 $\leq$ sin $\theta\leq 1$

$\Rightarrow$ -1 $\leq$ 5x $\leq$ 1

$\Rightarrow -\frac{1}{5}\leq$ x"$\frac{1}{5}$

$\Rightarrow$ x $\in\left[-\frac{1}{5},\frac{1}{5}\right]$

Thus, domain of $\sin^{-1}$ x is $\left[-\frac{1}{5},\frac{1}{5}\right]$.

27. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

28. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

29.

(d) A is false but R is true.

Explanation:

Assertion- $\sin^{-1}$ x should not be confused with $(\sin x)^{-1}$. Infact $(\sin x)^{-1}=\frac{1}{\sin x}$ and similarly for other trigonometric functions.

Reason- The value of an inverse trigonometric function that lies in the range of the principal branch, is called the principal value of that inverse trigonometric function. Hence, we can say that Assertion is false and Reason is true.

30.

(c) A is true but R is false.

Explanation:

$\sec^{-1}$ x is defined if x $\leq$ -1 or x $\geq$ 1. Hence, $\sec^{-1}$ 2x will be defined is x $\leq-\frac{1}{2}$ or x $\geq\frac{1}{2}$.

Hence, A is true.

The range of the function $\sec^{-1}$ x is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$, R is false.

31.

(c) A is true but R is false.

Explanation:

A is true but R is false.

32.

(c) A is true but R is false.

Explanation:

A is true but R is false.
Explanation:
A: The maximum value of $(\cos^{-1} x)^2$ is π^2 (true)
R: Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. (false)
The range of the principal value branch of $\cos^{-1}x$ is $[-\pi, \pi]$.

33.
(d) A is false but R is true.
Explanation:
A is false but R is true.
Explanation:
Assertion (A): All trigonometric functions have their inverses over their respective domains. (false)
As all trigonometric functions are periodic functions and hence manyone functions. Therefore trigonometric functions are not invertible over their respective domains.

Reason (R): The inverse of $\tan^{-1} x$ exists for some $x \in R$ (true)

34. State whether the given statement is True or False:

(i) **(b)** False
Explanation: {
False

(ii) **(a)** True
Explanation: {
True

(iii) **(a)** True
Explanation: {
True

(iv) **(a)** True
Explanation: {
True

(v) **(a)** True
Explanation: {
True

(vi) **(b)** False
Explanation: {
False

(vii) **(b)** False
Explanation: {
False