Solution

INVERSE TRIGONOMETRY

Class 12 - Mathematics

1.
(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
Explanation:

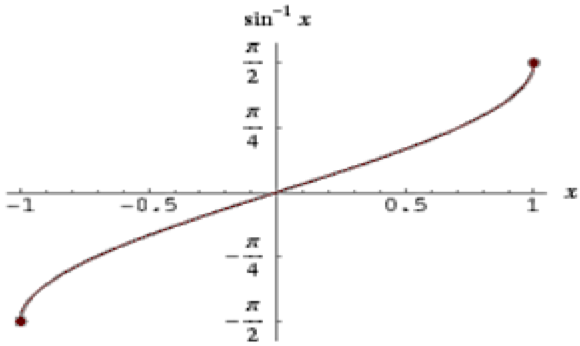
To Find: The range of $\sin^{-1}x$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sin^{-1}(x)$ can be obtained from the graph of
Y = sin x by interchanging x and y axes.i.e, if (a, b) is a point on Y = sin x then (b, a) is

The point on the function $y = \sin^{-1}(x)$

Below is the Graph of range of $\sin^{-1}(x)$



From the graph, it is clear that the range $\sin^{-1}(x)$ is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2.
(b) $\frac{2}{\sqrt{5}}$
Explanation:
$\frac{2}{\sqrt{5}}$

3.
(d) $-\frac{\pi}{3}$
Explanation:

$\sin^{-1} \sin (600)^o = \sin^{-1} [\sin(540^o + 60^o)]$

$= \sin^{-1} [\sin (3\pi + \frac{\pi}{3})]$

$= \sin^{-1} [-\sin \frac{\pi}{3}]$

$= \sin^{-1} [\sin\left(-\frac{\pi}{3}\right)]$

$= -\frac{\pi}{3}$

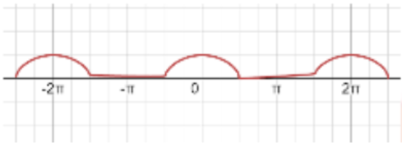
4. **(a)** $\frac{\pi}{4}$
Explanation:
$sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + cot^{-1}(3)$
$\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{3}\right)$ because $\frac{1}{2}.\frac{1}{3} < 1$
$\Rightarrow \tan^{-1}\left(\frac{\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right)$
$\Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

5.

(c) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

Explanation:

f(x) = $\sqrt{\cos x}$



According to graph of $\sqrt{\cos x}$ the domain is $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

6.

(c) [0, 1]

Explanation:

We have f(x) = $\cos^{-1}$ (2x - 1)

Since, -1 $\leq$ 2x - 1 $\leq$ 1

$\Rightarrow 0 \leq 2x \leq 2$

$\Rightarrow 0 \leq x \leq 1$

$\therefore x \in [0,1]$

7. **(a)** $\frac{\sqrt{x^2-1}}{x}$

Explanation:

if $\theta = \cos^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \cos\theta = \frac{1}{x} = \frac{Base}{Hyp.} \Rightarrow \tan\theta = \frac{Perp.}{Base} = \frac{\sqrt{x^2-1}}{x}$

8.

(b) $\sqrt{3}$

Explanation:

cot $\left[\frac{1}{2} \sin^{-1} \frac{\sqrt{3}}{2}\right]$

= cot $\left[\frac{1}{2} \times \frac{\pi}{3}\right]$ = $\cot^{-1}\left(\frac{\pi}{6}\right) = \sqrt{3}$

9. **(a)** 4

Explanation:

We have

$\sin^{-1}(x^2 - 7x + 12) = n\pi \Rightarrow x^2 - 7x + 12 = \sin(n\pi), n \in Z$

$\Rightarrow x^2 - 7x + 12 = 0 \Rightarrow x^2 - 3x - 4x + 12 = 0$

$\Rightarrow x(x - 3) - 4(x - 3) = 0 \Rightarrow (x - 3)(x - 4) = 0$

$\Rightarrow x = 3, 4$

10.

(d) -1 $\leq$ x $\leq$ 1

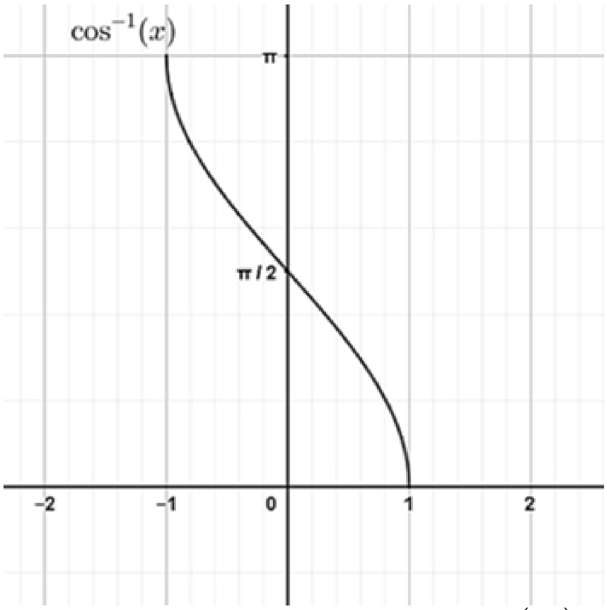
Explanation:

$sin^{-1}x$ is defined only for [-1,1]

$\therefore$ Domain of f(x) = $\sin^{-1}x$ is [-1, 1].

11. f(x) = $\cos^{-1}$ x.

principal branch of $\cos^{-1}x$ is $[0, \pi]$



12. The given inverse trignometric function is $\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

    Now,$\cos^{-1}\left(\frac{-1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$
    $=\left[\pi-\cos^{-1}\left(\frac{1}{2}\right)\right]+2\sin^{-1}\left(\frac{1}{2}\right)$
    $\left[\because\cos^{-1}(-x)=\pi-\cos^{-1}x;\forall x\in[-1,1]\right]$
    $=\left[\pi-\cos^{-1}\left(\cos\frac{\pi}{3}\right)\right]+2\sin^{-1}\left(\sin\frac{\pi}{6}\right)$
    $\left[\because\cos\frac{\pi}{3}=\frac{1}{2}\text{ and }\sin\frac{\pi}{6}=\frac{1}{2}\right]$
    $=\left[\pi-\frac{\pi}{3}\right]+2\times\frac{\pi}{6}$
    $\left[\begin{array}{c}\because\cos^{-1}(\cos\theta)=\theta;\forall\theta\in[0,\pi]\\ \text{and }\sin^{-1}(\sin\theta)=\theta;\forall\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\end{array}\right]$
    $=\frac{2\pi}{3}+\frac{\pi}{3}=\frac{2\pi+\pi}{3}=\pi$
13. Let $\cos^{-1}\left(-\frac{1}{2}\right)=\theta$
    $\cos\theta=\frac{-1}{2}$
    $\theta\in[0,\pi]$
    $\cos\theta=\cos\left(\pi-\frac{\pi}{3}\right)$
    $\theta=\frac{2\pi}{3}$
    Principal value is $\frac{2\pi}{3}$
14. $\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]$ = 1
15. Let $\cos^{-1}\left(\frac{-1}{2}\right)=y$
    $\Rightarrow\cos y=-\frac{1}{2}$
    $\Rightarrow\cos y=-\cos\frac{\pi}{3}$
    $\Rightarrow\cos y=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\frac{2\pi}{3}$

    Since, the principal value branch of $\cos^{-1}$ is $[0,\pi]$.
    Therefore, principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is $\frac{2\pi}{3}$.
16. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=y$
    $\Rightarrow\sec y=\frac{2}{\sqrt{3}}$
    $\Rightarrow\sec y=\sec\frac{\pi}{6}$

    Since, the principal value branch of $\sec^{-1}$ is $[0,\pi]$.
    Therefore, Principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.
17. $\cot^{-1}x$ represents an angle in (0, π) whose cotangent is x.
    Let $x=\cot^{-1}(\sqrt{3})$
    $\Rightarrow\cot x=\sqrt{3}=\cot\left(\frac{\pi}{6}\right)$

$\Rightarrow x = \frac{\pi}{6}$

$\therefore$ Principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

18. We know that $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$.

$\therefore \quad \tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

$= \tan^{-1}\left\{2\cos\left(2 \times \frac{\pi}{6}\right)\right\}$

$= \tan^{-1}\left(2\cos\frac{\pi}{3}\right) = \tan^{-1}(2 \times \frac{1}{2}) = \tan^{-1}1 = \frac{\pi}{4}$

19. Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$

$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$

$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$

$= \frac{\pi}{2} - \pi$

$= -\frac{\pi}{2}$

Therefore we have,

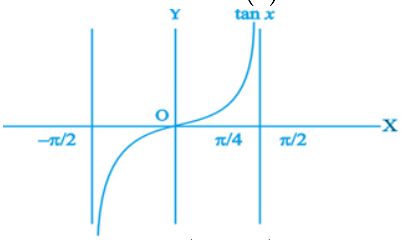
$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$

20. From Fig. we note that tan x is an increasing function in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan\frac{\pi}{4}$. This gives

tan 1 > 1

$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$

$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$



21. We have, $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\tan\left(\pi - \frac{\pi}{3}\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$ $\left[\because \tan^{-1}(-x) = -\tan^{-1}x\right]$

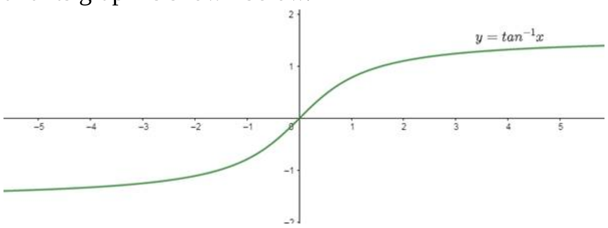
$= \tan^{-1}\tan\left(\frac{-\pi}{3}\right) = -\frac{\pi}{3}$ $\left[\because \tan^{-1}(\tan x) = x,\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)\right]$

Note: Remember that, $\tan^{-1}\left(\tan\frac{2\pi}{3}\right) \neq \frac{2\pi}{3}$

Since, $\tan^{-1}(\tan x) = x$, if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\frac{2\pi}{3} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

22. Principal value branch of $\tan^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and its graph is shown below.



23. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - \left[\pi - \sec^{-1}2\right]$

$= \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{2}\right)$

$= -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}$

24. We know that the range of the principal-value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Let $\tan^{-1}(\sqrt{3}) = \theta$, Then, we have,

$\tan\theta = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Hence, the principal value of $\tan^{-1}(\sqrt{3})$ is equal to $\frac{\pi}{3}$

25. Let us consider $\tan^{-1}(1) = x$ then we obtain

$\tan x = 1 = \tan\frac{\pi}{4}$

We know that range of the principle value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Thus, $\tan^{-1}(1) = \frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$

$\cos y = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

We know that range of the principle value branch of $\cos^{-1}$ is $[0, \pi]$

Thus, $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z$

$\sin z = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

We know that range of the principle value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Thus, $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Now,we have

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi+8\pi-2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

26. i. in $\triangle$BDA

$AB^2 = AD^2 + BD^2$

$= (30\sqrt{3})^2 + (30)^2$

$= (60)^2$

AB = 60 m

Now, $\sin\alpha = \frac{BD}{AB}$

$\sin\alpha = \frac{30}{60} = \frac{1}{2}$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

Again, In $\triangle$BDA

$\cos\alpha = \frac{AD}{AB} = \frac{30\sqrt{3}}{60} = \frac{\sqrt{3}}{2}$

$\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

ii. DC = AC - AD

DC = $40\sqrt{3} - 30\sqrt{3}$

DC = $10\sqrt{3}$ m

In $\triangle$BDC,

$\tan\beta = \frac{BD}{DC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$

$\beta = \tan^{-1}(\sqrt{3})$

iii. $\because \sin\alpha = \frac{1}{2}$

$\sin\alpha = \sin\ 30^\circ$

$\alpha = 30^\circ$

$\tan\beta = \sqrt{3}$

$\beta = 60^\circ$

Now, In $\triangle$ABC

$\angle ABC + \angle BAC + \angle ACB = 180^o$

$\angle ABC + 30^o + 60^o = 180^o$

$\angle ABC = 90^o$

$\angle ABC = \frac{\pi}{2}$

OR

Domain - [-1, 1] and range is $[0, \pi]$

27. i. In $\triangle$ABC,

tan A = $\frac{BC}{AB}$

tan A = $\frac{10}{20}$

tan A = $\frac{1}{2}$

$\angle A$ = tan^{-1} $\frac{1}{2}$

$\angle CAB$ = tan^{-1}$(\frac{1}{2})$

Now,

$\angle DAB = 2 \times \angle CAB$

= 2 $\times$ tan^{-1} $\frac{1}{2}$

Using 2tan^{-1}x = $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right)$

$= \tan^{-1}\left(\frac{1}{\frac{3}{4}}\right) = \tan^{-1}\left(\frac{4}{3}\right)$

$\angle DAB = \tan^{-1}\left(\frac{4}{3}\right)$

ii. $\angle EAB = 3 \times \angle CAB$

$= 3 \times \tan^{-1}\left(\frac{1}{2}\right)$

Using 3tan^{-1}x $= \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$\angle EAB = \tan^{-1}\left(\frac{3\times\frac{1}{2}-\left(\frac{1}{2}\right)^3}{1-3\left(\frac{1}{2}\right)^2}\right)$

$\angle EAB = \tan^{-1}\left(\frac{\frac{11}{8}}{\frac{1}{4}}\right) = \tan^{-1}\left(\frac{11}{2}\right)$

$\angle EAB$ = tan^{-1} $(\frac{11}{2})$

iii. In $\triangle$A'BC

tan A' = $\frac{BC}{A'B}$

tan A' = $\frac{10}{25}$

tan A' = $\frac{2}{5}$

$\angle A'$ = tan^{-1} $\frac{2}{5}$

$\angle CA'B$ = tan^{-1} $\frac{2}{5}$

Now, the difference

$\angle CAB - \angle CA'B$ = $\tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{2}{5}\right)$

Using tan^{-1}x - tan^{-1}y = tan^{-1} $\frac{x-y}{1+xy}$

= $\tan^{-1}\left[\frac{\frac{1}{2}-\frac{2}{5}}{1+\frac{1}{2}\times\frac{2}{5}}\right]$

= $\tan^{-1}\left[\frac{\frac{1}{10}}{\frac{6}{5}}\right] = \tan^{-1}\left(\frac{1}{12}\right)$

$\angle CAB - \angle CA'B =$ tan^{-1} $(\frac{1}{12})$

OR

Domain of tan^{-1}x is R i.e. all real numbers.

Range - R $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$