Atomic Structure

Derivation of the $\frac{e}{m}$ Equation

The $\frac{e}{m}$ ratio (charge-to-mass ratio) of an electron was first determined by J.J. Thomson through his experiments using cathode rays. Here's the detailed derivation of the $\frac{e}{m}$ equation.

Experimental Setup

Thomson's apparatus consisted of a cathode ray tube with an anode and a cathode. The tube was evacuated, and a high voltage was applied between the electrodes, creating a stream of electrons (cathode rays) moving from the cathode to the anode. These electrons were subjected to both electric and magnetic fields perpendicular to each other and to the path of the electrons.

Electric Field Analysis

When an electric field E is applied perpendicular to the motion of the electrons, the force on the electrons is given by:

$$F_e = eE$$

where:

- e is the charge of the electron.
- E is the electric field strength.

This force causes the electrons to be deflected.

Magnetic Field Analysis

When a magnetic field B is applied perpendicular to the motion of the electrons, the magnetic force on the electrons is given by:

$$F_m = evB$$

where:

- v is the velocity of the electrons.
- B is the magnetic field strength.

This force also causes the electrons to be deflected.

Balancing Forces

In Thomson's experiment, he adjusted the electric and magnetic fields such that their effects on the electron beam would cancel out, causing no net deflection. This happens when the electric force

equals the magnetic force:

$$eE = evB$$

Solving for the velocity v:

$$v = \frac{E}{B}$$

Determination of $\frac{e}{m}$

Next, by removing the electric field and measuring the radius r of the circular path of electrons in the magnetic field alone, we use the centripetal force relationship. The centripetal force required to keep the electron in a circular path is provided by the magnetic force:

$$F_m = \frac{mv^2}{r}$$

Substituting $F_m = evB$:

$$evB = \frac{mv^2}{r}$$

Solving for $\frac{e}{m}$:

$$\frac{e}{m} = \frac{v}{Br}$$

Using the velocity derived from balancing electric and magnetic forces:

$$v = \frac{E}{B}$$

Substitute v into the equation:

$$\frac{e}{m} = \frac{\frac{E}{B}}{Br} = \frac{E}{B^2 r}$$

Value of $\frac{e}{m}$

The experimentally determined value of $\frac{e}{m}$ for the electron is approximately:

$$\frac{e}{m} = 1.758820 \times 10^{11} \ \text{C/kg}$$

Summary

The derivation of the charge-to-mass ratio of an electron involves balancing the electric and magnetic forces acting on the electrons and then analyzing the resulting motion when only the magnetic field is present. The ratio $\frac{e}{m}$ is a fundamental property of the electron that was crucial in understanding the nature of cathode rays and the electron itself.