Dual Nature of Electrons

Here are six problems based on **de Broglie's concept**, including detailed steps for solving each one:

Problem 1: Calculate the de Broglie wavelength of an electron moving with a velocity of $2 \times 10^6 \, \mathrm{m/s}$.

Given:

- Velocity of electron, $v = 2 \times 10^6 \, \mathrm{m/s}$
- Mass of electron, $m = 9.1 \times 10^{-31} \, \mathrm{kg}$
- Planck's constant, $h = 6.626 \times 10^{-34} \, \mathrm{Js}$

Solution: We use the de Broglie wavelength formula:

$$\lambda = \frac{h}{mv}$$

Substitute the known values:

$$\lambda = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^6}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.82 \times 10^{-24}} \, \mathrm{m}$$

$$\lambda = 3.64 \times 10^{-10} \, \mathrm{m} = 0.364 \, \mathrm{nm}$$

Answer: The de Broglie wavelength is $0.364 \, \mathrm{nm}$.

Problem 2: Find the velocity of an electron if its de Broglie wavelength is $1.5 \, \mathrm{nm}$.

Given:

- de Broglie wavelength, $\lambda = 1.5 \times 10^{-9} \, \mathrm{m}$
- Mass of electron, $m = 9.1 \times 10^{-31} \, \mathrm{kg}$
- Planck's constant, $h = 6.626 \times 10^{-34} \, \mathrm{Js}$

Solution: Rearrange the de Broglie equation to solve for velocity:

$$v = \frac{h}{m\lambda}$$

Substitute the values:

$$v = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.5 \times 10^{-9}} \, \mathrm{m/s}$$

$$v = \frac{6.626 \times 10^{-34}}{1.365 \times 10^{-39}}\,\mathrm{m/s}$$

$$v = 4.85 \times 10^{5}\,\mathrm{m/s}$$

Answer: The velocity of the electron is $4.85 \times 10^{5}\,\mathrm{m/s}$.

Problem 3: What is the de Broglie wavelength of a proton moving with a velocity of $1 \times 10^{7}\,\mathrm{m/s}$?

Given:

- Velocity of proton, $v = 1 \times 10^{7}\,\mathrm{m/s}$
- Mass of proton, $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
- Planck's constant, $h = 6.626 \times 10^{-34}\,\mathrm{Js}$

Solution: Use the de Broglie wavelength formula:

$$\lambda = \frac{h}{mv}$$

Substitute the known values:

$$\lambda = \frac{6.626 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^{7}}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.67 \times 10^{-20}}\,\mathrm{m}$$

$$\lambda = 3.97 \times 10^{-14}\,\mathrm{m} = 39.7\,\mathrm{fm}$$

Answer: The de Broglie wavelength is $39.7\,\mathrm{fm}$ (femtometers).

Problem 4: Calculate the momentum of a particle with a de Broglie wavelength of $0.25\,\mathrm{nm}$.

Given:

- Wavelength, $\lambda = 0.25\,\mathrm{nm} = 0.25 \times 10^{-9}\,\mathrm{m}$
- Planck's constant, $h = 6.626 \times 10^{-34}\,\mathrm{Js}$

Solution: The de Broglie relation for momentum is:

$$p = \frac{h}{\lambda}$$

Substitute the known values:

$$p = \frac{6.626 \times 10^{-34}}{0.25 \times 10^{-9}}\,\mathrm{kg\,m/s}$$

$$p = \frac{6.626 \times 10^{-34}}{2.5 \times 10^{-10}}\,\mathrm{kg\,m/s}$$

$$p = 2.65 \times 10^{-24}\,\text{kg m/s}$$

Answer: The momentum is $2.65 \times 10^{-24}\,\text{kg m/s}$.

Problem 5: Determine the kinetic energy of an electron with a de Broglie wavelength of $0.2\,\text{nm}$.

Given:

- Wavelength, $\lambda = 0.2\,\text{nm} = 0.2 \times 10^{-9}\,\text{m}$
- Mass of electron, $m = 9.1 \times 10^{-31}\,\text{kg}$
- Planck's constant, $h = 6.626 \times 10^{-34}\,\text{Js}$

Solution: First, calculate the velocity from the de Broglie relation:

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.2 \times 10^{-9}}$$

$$v = 3.64 \times 10^{6}\,\text{m/s}$$

Now, calculate the kinetic energy:

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2} \times 9.1 \times 10^{-31} \times (3.64 \times 10^{6})^2$$

$$KE = \frac{1}{2} \times 9.1 \times 10^{-31} \times 1.325 \times 10^{13}$$

$$KE = 6.03 \times 10^{-18}\,\text{J}$$

Answer: The kinetic energy is $6.03 \times 10^{-18}\,\text{J}$.

Problem 6: Find the de Broglie wavelength of a tennis ball weighing $0.06\,\text{kg}$ moving at $30\,\text{m/s}$.

Given:

- Mass of tennis ball, $m = 0.06\,\text{kg}$
- Velocity, $v = 30\,\text{m/s}$
- Planck's constant, $h = 6.626 \times 10^{-34}\,\text{Js}$

Solution: Using the de Broglie equation:

$$\lambda = \frac{h}{mv}$$

Substitute the values:

$$\lambda = \frac{6.626 \times 10^{-34}}{0.06 \times 30}\,\mathrm{m}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.8}\,\mathrm{m}$$

$$\lambda = 3.68 \times 10^{-34}\,\mathrm{m}$$

Answer: The de Broglie wavelength is $3.68 \times 10^{-34}\,\mathrm{m}$, which is extremely small, confirming that macroscopic objects like tennis balls do not exhibit noticeable wave-like behavior.

These problems cover various aspects of the de Broglie hypothesis, showing how to calculate wavelengths, momentum, velocity, and kinetic energy of particles, and they help in understanding the wave-particle duality in practical scenarios.