

Angles in Radians Conversion

Example 2.1

Calculate the angle of (a) 1° (degree) (b) $1'$ (minute of arc or arcmin) and (c) $1''$ (second of arc or arc second) in radians. Use $360^\circ = 2\pi$ rad, $1^\circ = 60'$ and $1' = 60''$.

Solution:

(a) 1° to radians:

$$1^\circ = \frac{2\pi \text{ radians}}{360^\circ} = \frac{\pi}{180} \text{ radians}$$

(b) $1'$ to radians:

$$\text{Since } 1' = \frac{1^\circ}{60},$$

$$1' = \frac{\pi}{180 \times 60} = \frac{\pi}{10800} \text{ radians}$$

(c) $1''$ to radians:

$$\text{Since } 1'' = \frac{1'}{60},$$

$$1'' = \frac{\pi}{10800 \times 60} = \frac{\pi}{648000} \text{ radians}$$

Example 2.2

A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC up to B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle $\theta = 40^\circ$ (θ is known as 'parallax') estimate the distance of the tower C from his original position A.

Solution:

Given:

- Distance $AB = 100$ m
- Parallax angle $\theta = 40^\circ$

Using the formula for parallax distance:

$$\tan(\theta) = \frac{AB}{AC}$$

$$\tan(40^\circ) = \frac{100}{AC}$$

Using the value of $\tan(40^\circ) \approx 0.8391$:

$$0.8391 = \frac{100}{AC}$$

$$AC = \frac{100}{0.8391} \approx 119.14 \text{ m}$$

Example 2.3

The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is $1^\circ 54'$. Given the diameter of the Earth to be about 1.276×10^7 m, compute the distance of the moon from the Earth.

Solution:

Convert $1^\circ 54'$ to radians:

$$1^\circ 54' = 1 + \frac{54}{60} = 1.9^\circ$$

$$1.9^\circ = 1.9 \times \frac{\pi}{180} = 0.0332 \text{ radians}$$

Using the parallax formula:

$$\theta = \frac{d}{D}$$

$$0.0332 = \frac{1.276 \times 10^7 \text{ m}}{D}$$

$$D = \frac{1.276 \times 10^7}{0.0332} \approx 3.84 \times 10^8 \text{ m}$$

Example 2.4

The Sun's angular diameter is measured to be $1920''$. The distance D of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun?

Solution:

Convert $1920''$ to radians:

$$1920'' = 1920 \times \frac{\pi}{648000} = 9.32 \times 10^{-3} \text{ radians}$$

Using the angular diameter formula:

$$\theta = \frac{d}{D}$$

$$9.32 \times 10^{-3} = \frac{d}{1.496 \times 10^{11} \text{ m}}$$

$$d = 9.32 \times 10^{-3} \times 1.496 \times 10^{11} \approx 1.39 \times 10^9 \text{ m}$$

So, the diameter of the Sun is approximately 1.39×10^9 m.