Integrals :

Ex. 7.4.

① $\dfrac{1}{\sqrt{(2-x)^2+1}}$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+a\sqrt{x^2+a^2}\right|.$$

$$\int \frac{1}{\sqrt{(2-x)^2+1}} = \log\left|2-x+\sqrt{(2-x)^2+1}\right|$$

$$= \log\left|2-x+\sqrt{4-4x+x^2+1}\right|$$

$$= \log\left|2-x+\sqrt{x^2-4x+5}\right|.$$

② $\dfrac{1}{\sqrt{9-25x^2}} = \dfrac{1}{\sqrt{(3)^2-(5x)^2}}$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{2a}\sin^{-1}\left(\frac{x}{a}\right)+c.$$

$$\int \frac{dx}{\sqrt{(3)^2-(5x)^2}} = \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)+c.$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right)+c$$

Q.3 $\frac{3x}{1+2x^4}$

$= \frac{3x}{1+(\sqrt{2}x^2)^2}$

$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{dx \cdot 3x}{(1)^2+(\sqrt{2}x^2)^2} = \frac{3}{2\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}x^2}{1}\right) + C$

$= \frac{3}{2\sqrt{2}}\tan^{-1}(\sqrt{2}x^2) + C.$

Q.4 $\frac{x^2}{1-x^6}$

$\int \frac{x^2\,dx}{1-x^6} = \int \frac{x^2\,dx}{(1)^2-(x^3)^2}$

$t = x^3$

$dt = 3x^2\,dx.$

$= \frac{1}{3}\int \frac{dt}{1-t^2}$

$$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + c$$

$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + c.$$

Q.5 $\dfrac{1}{\sqrt{x^2+2x+2}}$

$$\int \frac{1}{\sqrt{x^2+2x+2}}\,dx = \int \frac{dx}{\sqrt{(x+1)^2+(1)^2}}$$

$t = x+1$

$dt = dx$

$$= \int \frac{dt}{\sqrt{t^2+1}}$$

$$= \log\left|t + \sqrt{t^2+1}\right| + c$$

$$= \log\left|(x+1) + \sqrt{(x+1)^2+1}\right| + c$$

$$= \log\left|(x+1) + \sqrt{x^2+2x+2}\right| + c.$$

Q.6 $\dfrac{\sec^2 x}{\sqrt{\tan^2 x + 4}}$

$\tan x = t$

$dt = \sec^2 x \cdot dx$

$$\int \frac{\sec^2 x \, dx}{\sqrt{\tan^2 x + 4}}$$

$$\int \frac{dt}{\sqrt{t^2 + (2)^2}}$$

$$= \log \left| t + \sqrt{t^2 + 4} \right| + C.$$

$$= \log \left| \sqrt{\tan x} + \sqrt{\tan^2 x + 4} \right| + C$$