

Integrals - Partial fractions

$$1.) \int \frac{x}{(x+1)(x+2)} dx$$

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$= \frac{A(x+2) + B(x+1)}{(x+1)(x+2)}$$

$$\frac{x}{(x+1)(x+2)} = \frac{A(x+2) + B(x+1)}{(x+1)(x+2)}$$

$$x = A(x+2) + B(x+1)$$

$$\begin{array}{l|l} x = -1 & x = -2 \\ -1 = A(1) & 2 = -1(B) \\ \boxed{A = -1} & \boxed{B = 2} \end{array}$$

$$\int \left(\frac{-1}{x+1} + \frac{2}{x+2} \right) dx$$

$$= \int \frac{1}{x+1} dx + 2 \int \frac{1}{x+2} dx$$

$$= \log|x+1| + 2 \log|x+2| + C$$

$$\log|x+2|^2 - \log|x+1| + C$$

$$\log \frac{(x+2)^2}{|x+1|} + C.$$

Q.2. $\int \frac{1}{x^2-9} dx$

$$\frac{1}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$\frac{1}{(x-3)(x+3)} = \frac{A(x+3) + B(x-3)}{(x-3)(x+3)}$$

$$1 = A(x+3) + B(x-3).$$

$$\begin{array}{l|l} x=3: & x=-3. \\ 9 = 6(A). & 1 = -6B. \\ \boxed{A = \frac{1}{6}} & \boxed{B = -\frac{1}{6}} \end{array}$$

$$\int \left(\frac{1}{6(x-3)} - \frac{1}{6(x+3)} \right) dx$$

$$\frac{1}{6} \log|x-3| - \frac{1}{6} \log|x+3| + C.$$

$$= \frac{1}{6} \log \frac{|x-3|}{|x+3|} + C.$$

Q.3

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$$x=2, \quad x=1,$$

$$6-1 = B(1)(-1) \quad \left| \quad 3-1 = A(-1)(-2) \right.$$

$$5 = -B$$

$$\boxed{B = -5}$$

$$2A = 2$$

$$\boxed{A = 1}$$

$$x=3,$$

$$9-1 = C(2)(1)$$

$$\boxed{C = 4}$$

$$\int \left(\frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3} \right) dx$$

$$\int \frac{1}{x-1} dx - \int \frac{5}{x-2} dx + \int \frac{4}{x-3} dx$$

$$\log|x-1| - 5 \log|x-2| + 4 \log|x-3| + C$$

Q.4 $\int \frac{x}{(x-1)(x-2)(x-3)} dx$

$$x = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$x=1$	$x=2$	$x=3$
$1 = A(-1)(-2)$	$2 = B(1)(-1)$	$3 = C(2)(1)$
$2A = 1$	$B = -2$	$C = \frac{3}{2}$
$A = \frac{1}{2}$		

$$\int \left(\frac{1}{2(x-1)} - \frac{2}{x-2} + \frac{3}{2(x-3)} \right) dx$$

$$\int \frac{1}{2(x-1)} dx - 2 \int \frac{1}{x-2} dx + \frac{3}{2} \int \frac{1}{x-3} dx$$

$$\frac{1}{2} \log|x-1| - 2 \log|x-2| + \frac{3}{2} \log|x-3| + C$$

Q.5

$$\int \frac{2x}{x^2+3x+2} dx$$

$$\frac{2x}{x^2+3x+2} = \frac{2x}{(x+1)(x+2)}$$

$$x^2+2x+x+2=0$$

$$x(x+2)+1(x+2)=0$$

$$(x+1)(x+2)=0$$

$$\frac{2x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$$2x = A(x+2) + B(x+1)$$

$$\begin{array}{l|l} x = -2 & x = -1 \\ -4 = B(-1) & -2 = A \\ \boxed{B = 4} & \boxed{A = -2} \end{array}$$

$$\int \left(\frac{-2}{x+1} + \frac{4}{x+2} \right) dx$$

$$\int \frac{-2}{x+1} dx + \int \frac{4}{x+2} dx$$

$$-2 \log|x+1| + 4 \log|x+2| + C$$

Q.6

$$\int \frac{1-x^2}{x(1-2x)}.$$

$$\frac{1-x^2}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$$

$$1-x^2 = A(1-2x) + Bx$$

$$x = \frac{1}{2}$$

$$1 - \frac{1}{4} = B \times \frac{1}{4}$$

$$\frac{4-1}{4} = \frac{B}{4}$$

$$\boxed{B = 3}$$

$$x = 0.$$

$$\boxed{A = 1}$$

$$\int \left(\frac{1}{x} + \frac{3}{1-2x} \right) dx$$

$$\int \frac{1}{x} dx + \int \frac{3}{1-2x} dx$$

$$\log x + 3 \log |1-2x| + C$$