# Integration by substitution

1) $\dfrac{1}{(x+x\log x)}$

$= \dfrac{1}{x(1+\log x)}$

let $1+\log x = t$

$\dfrac{1}{x}dx = dt$

$dx = x.dt$

$\displaystyle\int \frac{1}{x(1+\log x)}dx = \int \frac{1}{t}dt$

$= \log|t|+c \Rightarrow \log|1+\log x|+c$

2) $x\sqrt{x+2}$

let $x+2=t$

$dx = dt$

$= \displaystyle\int (t-2)\sqrt{t}\,dt = \int (t^{3/2} - 2t^{1/2})dt$

$= \left(\displaystyle\int t^{3/2} - \int 2t^{1/2}\right)dt = \dfrac{t^{5/2}}{5/2} - 2\dfrac{t^{3/2}}{3/2} + c$

$= \dfrac{2}{5}t^{5/2} - \dfrac{4}{3}t^{3/2} + c$

$= \dfrac{2}{5}(x+2)^{5/2} \cancel{+} - \dfrac{4}{3}(x+2)^{3/2} + c.$

3 $\frac{1}{x-\sqrt{x}}$

$= \frac{1}{\sqrt{x}(\sqrt{x}-1)}$

let $(\sqrt{x}-1) = t$

$\frac{1}{2\sqrt{x}} dx = dt$

$dx = 2\sqrt{x}.dt$

$= \int \frac{1}{\sqrt{x}(\sqrt{x}-1)} dx = \int \frac{2}{t} dt$

$= 2\log|t| + c \Rightarrow 2\log|\sqrt{x}-1| + c$

4 $\frac{x^2}{(2+3x^3)^3}$

let $2+3x^3 = t$

$9x^2 dx = dt$

$dx = \frac{dt}{9x^2}$

$= \frac{1}{9}\int \frac{dt}{t^3} = \frac{1}{9}\left[\frac{t^{-2}}{-2}\right] + c$

$= \frac{-1}{18}\left(\frac{1}{t^2}\right) + c$

$\Rightarrow \frac{-1}{18(2+3x^3)^2} + c.$

5) $\frac{x}{(9-4x^2)}$

$9-4x^2=t$

$-8x\,dx=dt$

$\int \frac{x}{9-4x^2}\,dx$

$= \frac{-1}{8}\int \frac{1}{t}\,dt = \frac{-1}{8}\log|t|+c$

$\Rightarrow \frac{-1}{8}\log|9-4x^2|+c$

6) $\frac{e^{\tan^{-1}x}}{1+x^2}$

let $\tan^{-1}x = t$

$\frac{1}{1+x^2}\,dx = dt$

$dx = (1+x^2)\,dt$

$\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx = \int \frac{e^{\tan^{-1}x}}{t}\,t\,d\cancel{t}$

$= \int e^{\tan^{-1}x}\,dt \Rightarrow \underline{e^{\tan^{-1}x}+c}.$