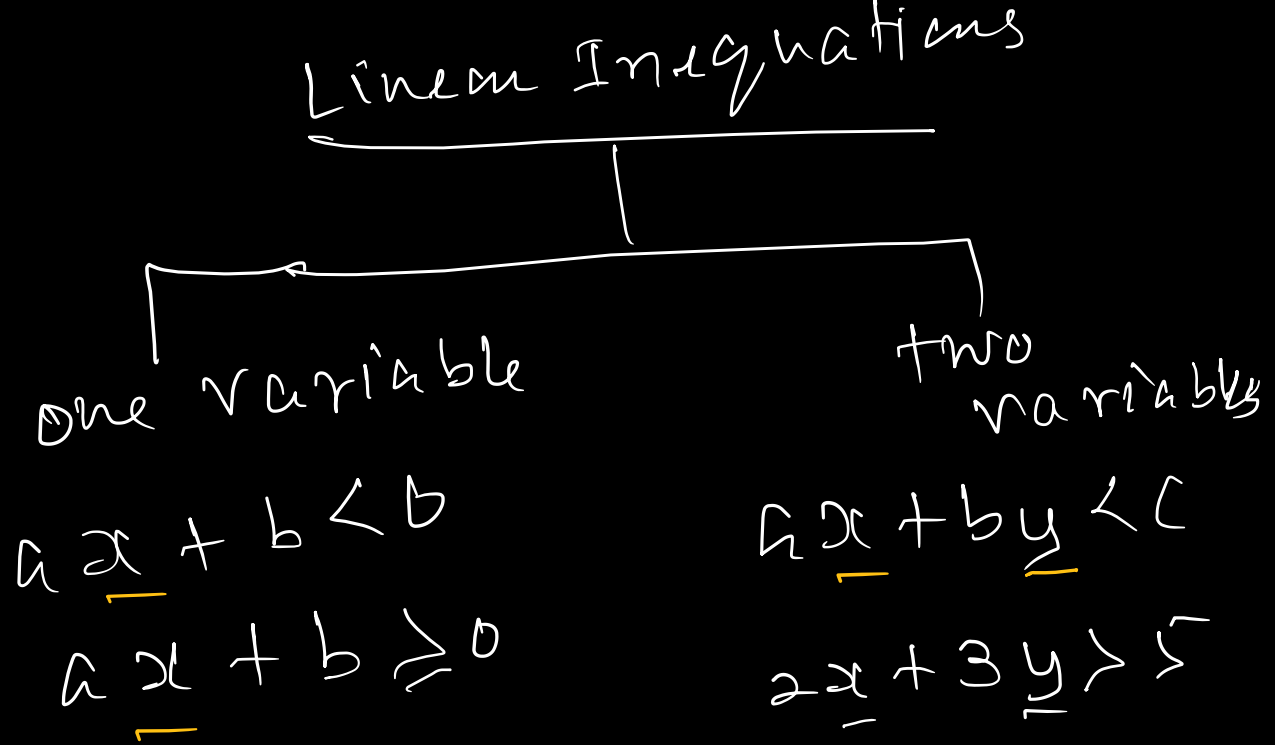
Linear Inequalities

$<$, $>$, $\leq$, $\geq$

$3x + 4 > 0$

$2x + 3y < 5$

$5x + 4 \geq 12$



Rules for solving an inequality

1. Equal numbers may be added to both sides without affecting the sign of inequality.
2. Equal numbers may be subtracted from both sides without affecting the sign of inequality.
3. Both sides of an inequality can be multiplied by the same positive number. But when both sides are multiplied by a negative number then sign of inequality is reversed.
4. Both sides of an inequality can be divided by the same positive number, but when both sides are divided by a negative number then the sign of inequality is reversed.

Solve $18x < 150$ when i) x is a natural number. Ii) x is an integer.

$18x < 150$

$\times \frac{1}{18} \qquad \times \frac{1}{18}$

$\frac{1}{18} \times 18x < 150 \times \frac{1}{18}$

$x < \frac{25}{3}$

$x < 8\frac{1}{3}$

(i) $x \in N$

$x = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(ii) $x = \{\ldots -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Solve $3x + 17 \leq 2(1 - x)$

$3x + 17 \leq 2(1 - x)$

$3x + 17 \leq 2 - 2x$

$+2x \qquad +2x$

$5x + 17 \leq 2$

$-17 \qquad -17$

$5x \leq -15$

$\frac{1}{5} \times \qquad \times \frac{1}{5}$

$x \leq -3$

Solution Set $(-\infty, -3]$

$(-\infty, \ldots -5, -4, -3]$

Solve $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$

Show the graph of the solution on number line.

$\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$

$-\frac{4x}{3} \qquad -\frac{4x}{3}$

$\frac{2x-3}{4} - \frac{4x}{3} + 9 \geq 3$

$-9 \qquad -9$

$\frac{2x-3}{4} - \frac{4x}{3} \geq -6$

$\frac{6x-9-16x}{12} \geq -6$

$\times 12 \qquad \times 12$

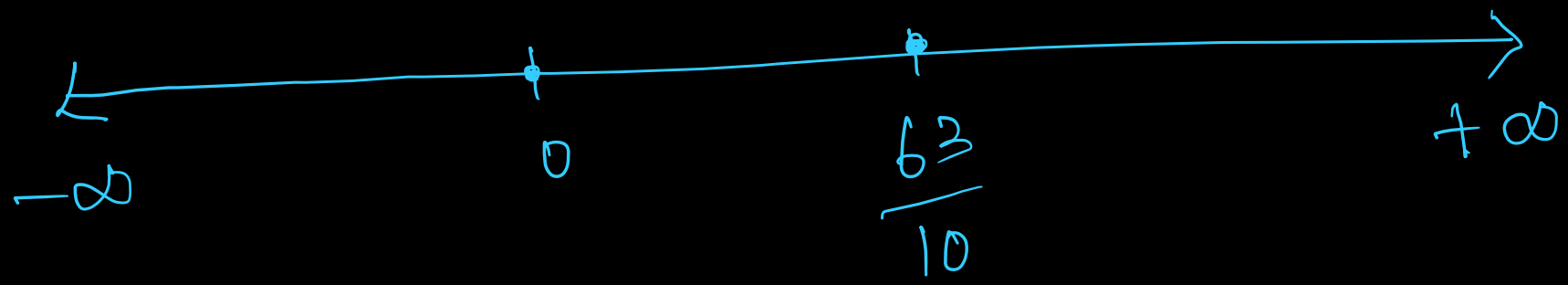
$-10x - 9 \geq -72$

$+9 \qquad +9$

$-10x \geq -63$

$\times\left(-\frac{1}{10}\right) \qquad \times\left(-\frac{1}{10}\right)$

$$x \leq \frac{63}{10}$$

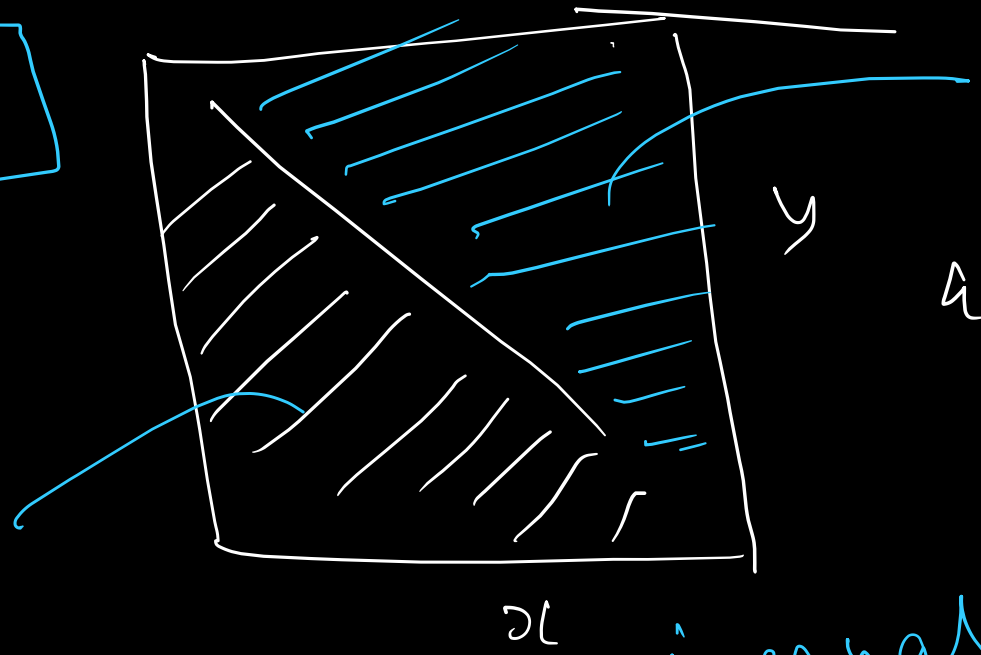


$$\frac{1}{2}\left(\frac{3x}{5}+4\right) \geq \frac{1}{3}(x-6)$$

$$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$$

Graphical Solutions of Linear Inequalities in Two Variables

$ax + by = c$



$ax + by \leq c$

$ax + by \geq c$

Half spaces of inequalities

Solve $5x + 2y \leq 10$

$5x + 2y = 10$

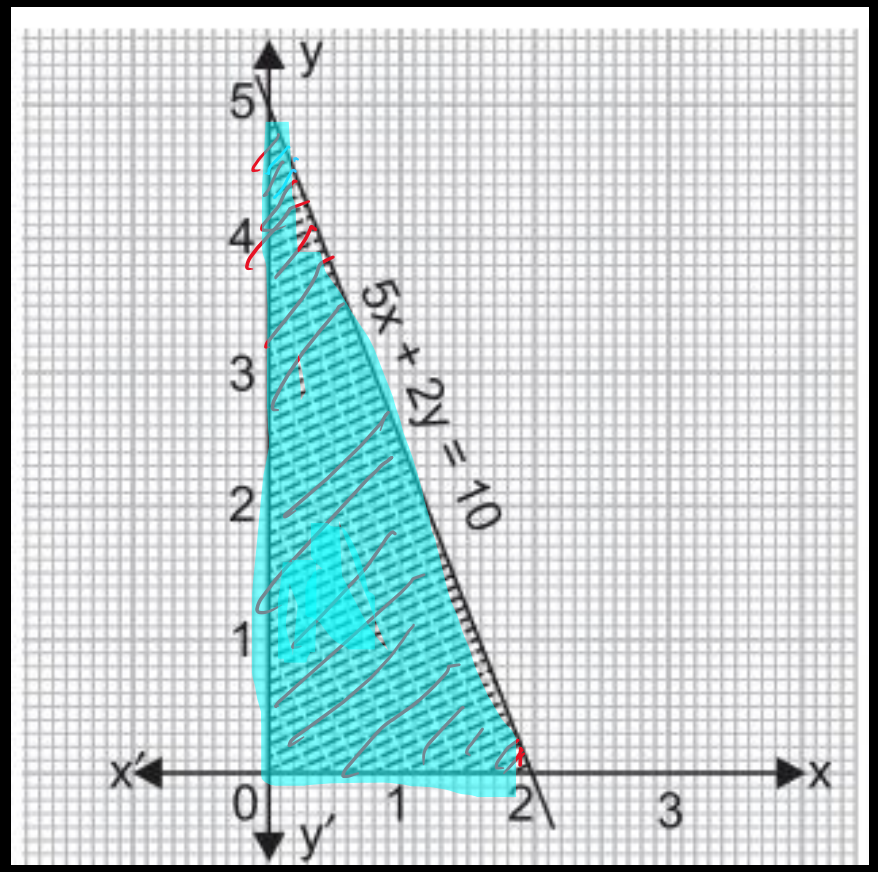
x	0	2
y	5	0

$(0,0)$

$(5 \times 0) + 2(0) \leq 10$

$0 \leq 10$

True



$$x + y \geq 5$$

$$x + y = 5$$

x	0	5
y	5	0

$(0,0)$

$0 + 0 \geq 5$

Not true

Not pointed towards the origin $(0,0)$