



Chapter Ten

WAVE OPTICS

10.1 INTRODUCTION

In 1637 Descartes gave the corpuscular model of light and derived Snell's law. It explained the laws of reflection and refraction of light at an interface. The corpuscular model predicted that if the ray of light (on refraction) bends towards the normal then the speed of light would be greater in the second medium. This corpuscular model of light was further developed by Isaac Newton in his famous book entitled *OPTICKS* and because of the tremendous popularity of this book, the corpuscular model is very often attributed to Newton.

In 1678, the Dutch physicist Christiaan Huygens put forward the wave theory of light – it is this wave model of light that we will discuss in this chapter. As we will see, the wave model could satisfactorily explain the phenomena of reflection and refraction; however, it predicted that on refraction if the wave bends towards the normal then the speed of light would be less in the second medium. This is in contradiction to the prediction made by using the corpuscular model of light. It was much later confirmed by experiments where it was shown that the speed of light in water is less than the speed in air confirming the prediction of the wave model; Foucault carried out this experiment in 1850.

The wave theory was not readily accepted primarily because of Newton's authority and also because light could travel through vacuum

and it was felt that a wave would always require a medium to propagate from one point to the other. However, when Thomas Young performed his famous interference experiment in 1801, it was firmly established that light is indeed a wave phenomenon. The wavelength of visible light was measured and found to be extremely small; for example, the wavelength of yellow light is about $0.6 \mu\text{m}$. Because of the smallness of the wavelength of visible light (in comparison to the dimensions of typical mirrors and lenses), light can be assumed to approximately travel in straight lines. This is the field of geometrical optics, which we had discussed in the previous chapter. Indeed, the branch of optics in which one completely neglects the finiteness of the wavelength is called geometrical optics and a ray is defined as the path of energy propagation in the limit of wavelength tending to zero.

After the interference experiment of Young in 1801, for the next 40 years or so, many experiments were carried out involving the interference and diffraction of lightwaves; these experiments could only be satisfactorily explained by assuming a wave model of light. Thus, around the middle of the nineteenth century, the wave theory seemed to be very well established. The only major difficulty was that since it was thought that a wave required a medium for its propagation, how could light waves propagate through vacuum. This was explained when Maxwell put forward his famous electromagnetic theory of light. Maxwell had developed a set of equations describing the laws of electricity and magnetism and using these equations he derived what is known as the wave equation from which he *predicted* the existence of electromagnetic waves*. From the wave equation, Maxwell could calculate the speed of electromagnetic waves in free space and he found that the theoretical value was very close to the measured value of speed of light. From this, he propounded that *light must be an electromagnetic wave*. Thus, according to Maxwell, light waves are associated with changing electric and magnetic fields; changing electric field produces a time and space varying magnetic field and a changing magnetic field produces a time and space varying electric field. The changing electric and magnetic fields result in the propagation of electromagnetic waves (or light waves) even in vacuum.

In this chapter we will first discuss the original formulation of the *Huygens principle* and derive the laws of reflection and refraction. In Sections 10.4 and 10.5, we will discuss the phenomenon of interference which is based on the principle of superposition. In Section 10.6 we will discuss the phenomenon of diffraction which is based on Huygens-Fresnel principle. Finally in Section 10.7 we will discuss the phenomenon of polarisation which is based on the fact that the light waves are *transverse electromagnetic waves*.

* Maxwell had predicted the existence of electromagnetic waves around 1855; it was much later (around 1890) that Heinrich Hertz produced radiowaves in the laboratory. J.C. Bose and G. Marconi made practical applications of the *Hertzian waves*

DOES LIGHT TRAVEL IN A STRAIGHT LINE?

Light travels in a straight line in Class VI; it does not do so in Class XII and beyond! Surprised, aren't you?

In school, you are shown an experiment in which you take three cardboards with pinholes in them, place a candle on one side and look from the other side. If the flame of the candle and the three pinholes are in a straight line, you can see the candle. Even if one of them is displaced a little, you cannot see the candle. *This proves, so your teacher says, that light travels in a straight line.*

In the present book, there are two consecutive chapters, one on ray optics and the other on wave optics. Ray optics is based on rectilinear propagation of light, and deals with mirrors, lenses, reflection, refraction, etc. Then you come to the chapter on wave optics, and you are told that light travels as a wave, that it can bend around objects, it can diffract and interfere, etc.

In optical region, light has a wavelength of about half a micrometre. If it encounters an obstacle of about this size, it can bend around it and can be seen on the other side. Thus a micrometre size obstacle will not be able to stop a light ray. If the obstacle is much larger, however, light will not be able to bend to that extent, and will not be seen on the other side.

This is a property of a wave in general, and can be seen in sound waves too. The sound wave of our speech has a wavelength of about 50 cm to 1 m. If it meets an obstacle of the size of a few metres, it bends around it and reaches points behind the obstacle. But when it comes across a larger obstacle of a few hundred metres, such as a hillock, most of it is reflected and is heard as an echo.

Then what about the primary school experiment? What happens there is that when we move any cardboard, the displacement is of the order of a few millimetres, which is much larger than the wavelength of light. Hence the candle cannot be seen. If we are able to move one of the cardboards by a micrometer or less, light will be able to diffract, and the candle will still be seen.

One could add to the first sentence in this box: *It learns how to bend as it grows up!*

10.2 HUYGENS PRINCIPLE

We would first define a wavefront: when we drop a small stone on a calm pool of water, waves spread out from the point of impact. Every point on the surface starts oscillating with time. At any instant, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points, which oscillate in phase is called a *wavefront*; thus a *wavefront is defined as a surface of constant phase*. The speed with which the wavefront moves outwards from the source is called the speed of the wave. The energy of the wave travels in a direction perpendicular to the wavefront.

If we have a point source emitting waves uniformly in all directions, then the locus of points which have the same amplitude and vibrate in the same phase are spheres and we have what is known as a *spherical wave* as shown in Fig. 10.1(a). At a large distance from the source, a

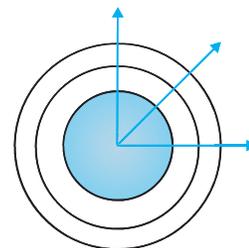


FIGURE 10.1 (a) A diverging spherical wave emanating from a point source. The wavefronts are spherical.

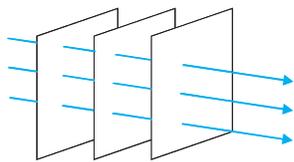


FIGURE 10.1 (b) At a large distance from the source, a small portion of the spherical wave can be approximated by a plane wave.

small portion of the sphere can be considered as a plane and we have what is known as a *plane wave* [Fig. 10.1(b)].

Now, if we know the shape of the wavefront at $t = 0$, then Huygens principle allows us to determine the shape of the wavefront at a later time τ . Thus, Huygens principle is essentially a geometrical construction, which given the shape of the wavefront at any time allows us to determine the shape of the wavefront at a later time. Let us consider a diverging wave and let F_1F_2 represent a portion of the spherical wavefront at $t = 0$ (Fig. 10.2). Now, according to Huygens principle, *each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.*

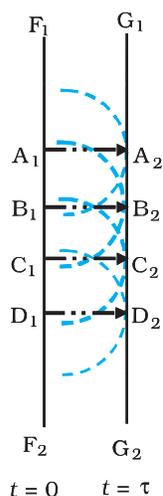


FIGURE 10.3 Huygens geometrical construction for a plane wave propagating to the right. F_1F_2 is the plane wavefront at $t = 0$ and G_1G_2 is the wavefront at a later time τ . The lines A_1A_2 , B_1B_2 ... etc., are normal to both F_1F_2 and G_1G_2 and represent rays.

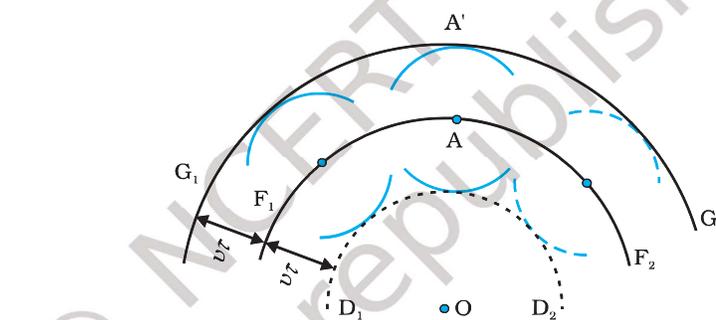


FIGURE 10.2 F_1F_2 represents the spherical wavefront (with O as centre) at $t = 0$. The envelope of the secondary wavelets emanating from F_1F_2 produces the forward moving wavefront G_1G_2 . The backwave D_1D_2 does not exist.

Thus, if we wish to determine the shape of the wavefront at $t = \tau$, we draw spheres of radius $v\tau$ from each point on the spherical wavefront where v represents the speed of the waves in the medium. If we now draw a common tangent to all these spheres, we obtain the new position of the wavefront at $t = \tau$. The new wavefront shown as G_1G_2 in Fig. 10.2 is again spherical with point O as the centre.

The above model has one shortcoming: we also have a backwave which is shown as D_1D_2 in Fig. 10.2. Huygens argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction; by making this adhoc assumption, Huygens could explain the absence of the backwave. However, this adhoc assumption is not satisfactory and the absence of the backwave is really justified from more rigorous wave theory.

In a similar manner, we can use Huygens principle to determine the shape of the wavefront for a plane wave propagating through a medium (Fig. 10.3).

10.3 REFRACTION AND REFLECTION OF PLANE WAVES USING HUYGENS PRINCIPLE

10.3.1 Refraction of a plane wave

We will now use Huygens principle to derive the laws of refraction. Let PP' represent the surface separating medium 1 and medium 2, as shown in Fig. 10.4. Let v_1 and v_2 represent the speed of light in medium 1 and medium 2, respectively. We assume a plane wavefront AB propagating in the direction $A'A$ incident on the interface at an angle i as shown in the figure. Let τ be the time taken by the wavefront to travel the distance BC . Thus,

$$BC = v_1 \tau$$

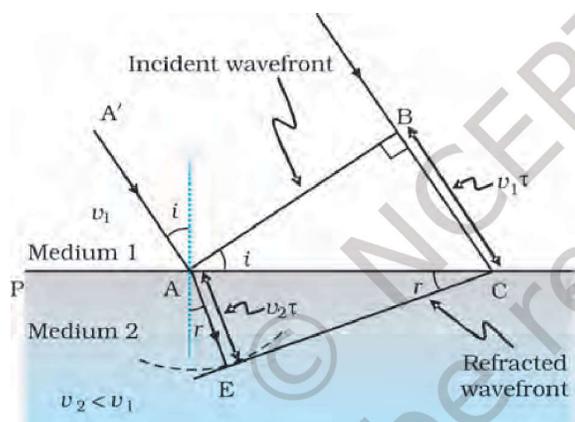


FIGURE 10.4 A plane wave AB is incident at an angle i on the surface PP' separating medium 1 and medium 2. The plane wave undergoes refraction and CE represents the refracted wavefront. The figure corresponds to $v_2 < v_1$ so that the refracted waves bends towards the normal.

In order to determine the shape of the refracted wavefront, we draw a sphere of radius $v_2 \tau$ from the point A in the second medium (the speed of the wave in the second medium is v_2). Let CE represent a tangent plane drawn from the point C on to the sphere. Then, $AE = v_2 \tau$ and CE would represent the refracted wavefront. If we now consider the triangles ABC and AEC , we readily obtain

$$\sin i = \frac{BC}{AC} = \frac{v_1 \tau}{AC} \quad (10.1)$$

and

$$\sin r = \frac{AE}{AC} = \frac{v_2 \tau}{AC} \quad (10.2)$$

where i and r are the angles of incidence and refraction, respectively.



Christiaan Huygens (1629 - 1695) Dutch physicist, astronomer, mathematician and the founder of the wave theory of light. His book, *Treatise on light*, makes fascinating reading even today. He brilliantly explained the double refraction shown by the mineral calcite in this work in addition to reflection and refraction. He was the first to analyse circular and simple harmonic motion and designed and built improved clocks and telescopes. He discovered the true geometry of Saturn's rings.

CHRISTIAAN HUYGENS (1629 - 1695)

Thus we obtain

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} \quad (10.3)$$

From the above equation, we get the important result that if $r < i$ (i.e., if the ray bends toward the normal), the speed of the light wave in the second medium (v_2) will be less than the speed of the light wave in the first medium (v_1). This prediction is opposite to the prediction from the corpuscular model of light and as later experiments showed, the prediction of the wave theory is correct. Now, if c represents the speed of light in vacuum, then,

$$n_1 = \frac{c}{v_1} \quad (10.4)$$

and

$$n_2 = \frac{c}{v_2} \quad (10.5)$$

are known as the refractive indices of medium 1 and medium 2, respectively. In terms of the refractive indices, Eq. (10.3) can be written as

$$n_1 \sin i = n_2 \sin r \quad (10.6)$$

This is the *Snell's law of refraction*. Further, if λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1 then the distance AE will be equal to λ_2 (because if the crest from B has reached C in time τ , then the crest from A should have also reached E in time τ); thus,

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1}{v_2}$$

or

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2} \quad (10.7)$$

The above equation implies that when a wave gets refracted into a denser medium ($v_1 > v_2$) the wavelength and the speed of propagation decrease but the *frequency* $\nu (= v/\lambda)$ remains the same.

10.3.2 Refraction at a rarer medium

We now consider refraction of a plane wave at a rarer medium, i.e., $v_2 > v_1$. Proceeding in an exactly similar manner we can construct a refracted wavefront as shown in Fig. 10.5. The angle of refraction will now be greater than angle of incidence; however, we will still have $n_1 \sin i = n_2 \sin r$. We define an angle i_c by the following equation

$$\sin i_c = \frac{n_2}{n_1} \quad (10.8)$$

Thus, if $i = i_c$ then $\sin r = 1$ and $r = 90^\circ$. Obviously, for $i > i_c$, there can not be any refracted wave. The angle i_c is known as the *critical angle* and for all angles of incidence greater than the critical angle, we will not have

any refracted wave and the wave will undergo what is known as *total internal reflection*. The phenomenon of total internal reflection and its applications was discussed in Section 9.4.

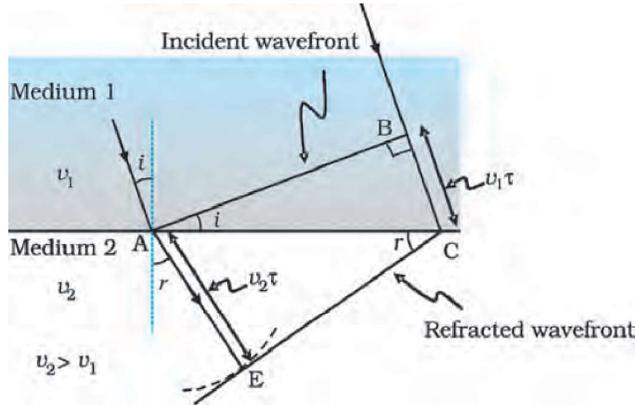


FIGURE 10.5 Refraction of a plane wave incident on a rarer medium for which $v_2 > v_1$. The plane wave bends away from the normal.

10.3.3 Reflection of a plane wave by a plane surface

We next consider a plane wave AB incident at an angle i on a reflecting surface MN. If v represents the speed of the wave in the medium and if τ represents the time taken by the wavefront to advance from the point B to C then the distance

$$BC = v\tau$$

In order to construct the reflected wavefront we draw a sphere of radius $v\tau$ from the point A as shown in Fig. 10.6. Let CE represent the tangent plane drawn from the point C to this sphere. Obviously

$$AE = BC = v\tau$$

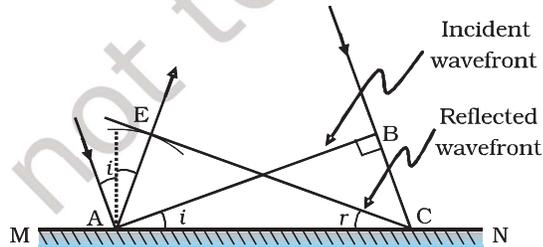


FIGURE 10.6 Reflection of a plane wave AB by the reflecting surface MN. AB and CE represent incident and reflected wavefronts.

If we now consider the triangles EAC and BAC we will find that they are congruent and therefore, the angles i and r (as shown in Fig. 10.6) would be equal. This is the *law of reflection*.

Once we have the laws of reflection and refraction, the behaviour of prisms, lenses, and mirrors can be understood. These phenomena were

discussed in detail in Chapter 9 on the basis of rectilinear propagation of light. Here we just describe the behaviour of the wavefronts as they undergo reflection or refraction. In Fig. 10.7(a) we consider a plane wave passing through a thin prism. Clearly, since the speed of light waves is less in glass, the lower portion of the incoming wavefront (which travels through the greatest thickness of glass) will get delayed resulting in a tilt in the emerging wavefront as shown in the figure. In Fig. 10.7(b) we consider a plane wave incident on a thin convex lens; the central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most. The emerging wavefront has a depression at the centre and therefore the wavefront becomes spherical and converges to the point F which is known as the *focus*. In Fig. 10.7(c) a plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focal point F . In a similar manner, we can understand refraction and reflection by concave lenses and convex mirrors.

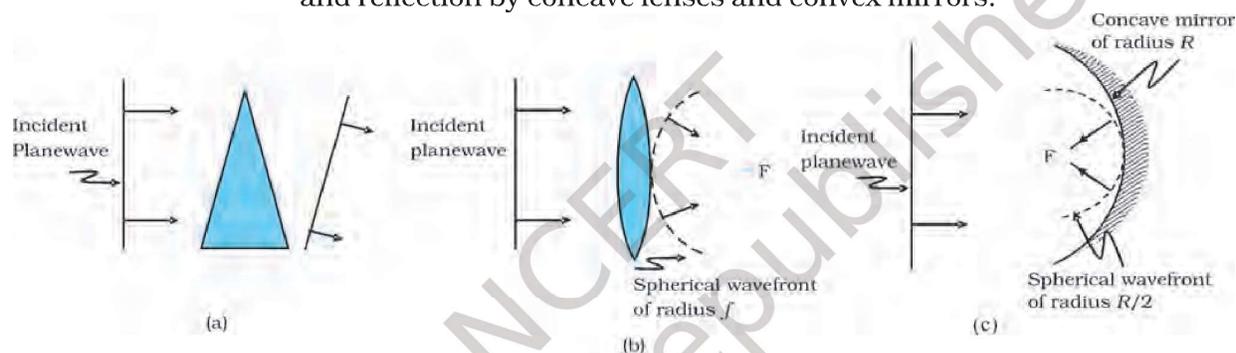


FIGURE 10.7 Refraction of a plane wave by (a) a thin prism, (b) a convex lens. (c) Reflection of a plane wave by a concave mirror.

From the above discussion it follows that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray. For example, when a convex lens focusses light to form a real image, although the ray going through the centre traverses a shorter path, but because of the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.

10.3.4 The doppler effect

We should mention here that one should be careful in constructing the wavefronts if the source (or the observer) is moving. For example, if there is no medium and the source moves away from the observer, then later wavefronts have to travel a greater distance to reach the observer and hence take a longer time. The time taken between the arrival of two successive wavefronts is hence longer at the observer than it is at the source. Thus, when the source moves away from the observer the frequency as measured by the source will be smaller. This is known as the *Doppler effect*. Astronomers call the increase in wavelength due to doppler effect as *red shift* since a wavelength in the middle of the visible region of the spectrum moves towards the red end of the spectrum. When waves are received from a source moving towards the observer, there is an apparent decrease in wavelength, this is referred to as *blue shift*.