Moving Charges

Important Derivations in "Moving Charges and Magnetism"

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1. Magnetic Force on a Moving Charge

Expression:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

Derivation Steps:

1. A charged particle with charge q moving with velocity $\vec{v}$ enters a region with magnetic field $\vec{B}$.
2. The magnetic force is given by the Lorentz force equation:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

3. For the case of only magnetic fields ($\vec{E} = 0$), the force becomes:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

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2. Magnetic Field on the Axis of a Circular Current Loop

Expression:

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

Derivation Steps:

1. Consider a circular loop of radius R carrying current I, and let x be the distance of a point on the axis from the center.
2. From the Biot-Savart law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{l} \times \hat{r}}{r^2}$$

3. The contributions to the field from diametrically opposite elements add up due to symmetry, and the net field is along the axis.
4. The magnetic field at a distance x from the center on the axis is integrated, yielding:

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

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3. Magnetic Field Due to a Straight Current-Carrying Wire

Expression:

$$B = \frac{\mu_0 I}{2\pi r}$$

Derivation Steps:

1. Consider a long straight wire carrying current I, and a point P at a perpendicular distance r.
2. Using the Biot-Savart law:

$$dB = \frac{\mu_0}{4\pi} \frac{I \, dl \sin\theta}{r^2}$$

3. Integrate along the length of the wire for limits from $-\infty$ to $+\infty$.
4. The resultant magnetic field at distance r is:

$$B = \frac{\mu_0 I}{2\pi r}$$

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4. Magnetic Field at the Center of a Circular Current Loop

Expression:

$$B = \frac{\mu_0 I}{2R}$$

Derivation Steps:

1. Consider a circular loop of radius R carrying current I.
2. Using the Biot-Savart law for an element $d\vec{l}$, symmetry ensures contributions to the field are along the center.
3. The field at the center simplifies to:

$$B = \frac{\mu_0 I}{2R}$$

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5. Ampere's Circuital Law

Expression:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

Derivation Steps:

1. Consider a closed loop around a current-carrying conductor.
2. Using symmetry, the magnetic field $\vec{B}$ is tangential, and $d\vec{l}$ lies along the loop.
3. The integral simplifies as:

$$\oint \vec{B} \cdot d\vec{l} = B \oint dl = B \cdot 2\pi r$$

4. Substituting B:

$$B \cdot 2\pi r = \mu_0 I$$

5. Thus, Ampere's circuital law is:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

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6. Magnetic Field Due to a Solenoid

Expression:

$$B = \mu_0 n I$$

Derivation Steps:

1. Consider a solenoid with n turns per unit length and current I.
2. Apply Ampere's circuital law to a rectangular loop:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

3. Enclosed current is $I_{\text{enc}} = nI \cdot l$, where l is the length of the solenoid.
4. The field inside is uniform, giving:

$$B = \mu_0 n I$$

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7. Magnetic Field Due to a Toroid

Expression:

$$B = \frac{\mu_0 N I}{2\pi r}$$

Derivation Steps:

1. Consider a toroid with N turns, carrying current I, and radius r.
2. Using Ampere's law for a circular path within the toroid:

$$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r = \mu_0 N I$$

3. Solve for B:

$$B = \frac{\mu_0 N I}{2\pi r}$$

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8. Radius of Charged Particle in Magnetic Field

Expression:

$$r = \frac{mv}{qB}$$

Derivation Steps:

1. A charged particle of mass m, charge q, and velocity v moves in a circular path in a uniform magnetic field B.
2. Centripetal force is provided by the magnetic force:

$$\frac{mv^2}{r} = qvB$$

3. Solve for r:

$$r = \frac{mv}{qB}$$

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9. Cyclotron Frequency

Expression:

$$f = \frac{qB}{2\pi m}$$

Derivation Steps:

1. From the circular motion of a charged particle:

$$T = \frac{2\pi r}{v}, \quad r = \frac{mv}{qB}$$

2. Period $T = \frac{2\pi m}{qB}$.
3. Frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$
