1. Explain Magnetic dipole moment of a bar magnet
2. Explain the properties of magnetic field lines.
3. Derive the magnetic field in the direction of magnetic field.
4. Derive the magnetic field of a bar magnet in the equatorial axis.
5. Show that the solenoid is equivalent to a bar magnet.
6. Derive the force experience by a magnetic dipole in a uniform magnetic field.
7. Derive the potential energy of dipole kept in a uniform magnetic field.

Magnetic dipole moment of bar magnet (*m*)

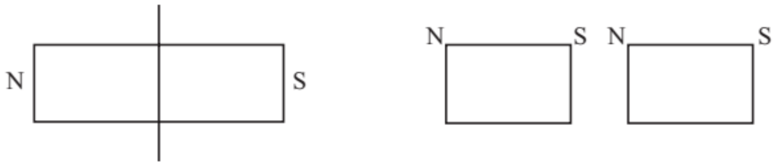
$$m = q_m \times 2l$$

where q_m = pole strength

$2l$ = dipole length.

Direction of magnet dipole moment is from south pole to north pole.

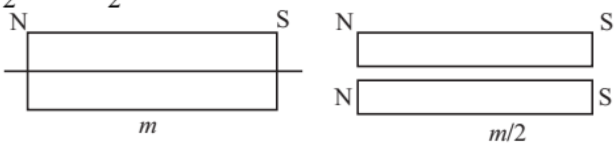
If a bar magnet is cut into two equal halves along a line ⊥ to its axis.



Dipole moment (original) 'm' $= q_m \times 2l$

Dipole moment of each part = $m' = q_m l = \frac{m}{2}$

It a bar magnet is cut into two equal parts longitudinally i.e., along axis of magnet then new dipole moment $m' = \frac{q_m}{2} \times 2l = \frac{m}{2}$

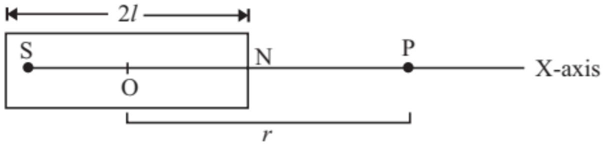


Magnetic Field Lines Properties :

1. They are closed curves which start in air from north pole and enter into south pole.
2. They never cross each other.
3. They come out/enter into a surface of magnet normally.
4. Relative closeness of field lines gives strength of the magnetic field at that place.

Magnetic Field of a Bar Magnet at an Axial Point

- Consider a bar magnet of length $2l$ and pole strength q_m.



$$\vec{B}_S = \frac{\mu_o}{4\pi}\frac{q_m}{(SP)^2} = \frac{\mu_o}{4\pi}\frac{q_m}{(r+l)^2}(-\hat{i})$$

$$\vec{B}_N = -\frac{\mu_o}{4\pi}\frac{q_m}{(NP)^2} = \frac{\mu_o}{4\pi}\frac{q_m}{(r-l)^2}(\hat{i})$$

$$\vec{B}_{net} = \vec{B}_S + \vec{B}_N = \frac{\mu_o}{4\pi} q_m(\hat{i})\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$= \frac{\mu_o q_m(\hat{i})}{4\pi}\left[\frac{4rl}{(r^2-l^2))^2}\right] = \frac{\mu_o 2mr\,(\hat{i})}{4\pi(r^2-l^2)^2}$$

Magnetic field is in direction of magnetic moment.

- Special case: If $r >>> l$

$$\vec{B}_{net} = \frac{\mu_o 2mr}{4\pi r^4}(\hat{i}) = \frac{\mu_o 2m}{4\pi r^3}(\hat{i})$$

$$\therefore B \propto \frac{1}{r^3}$$

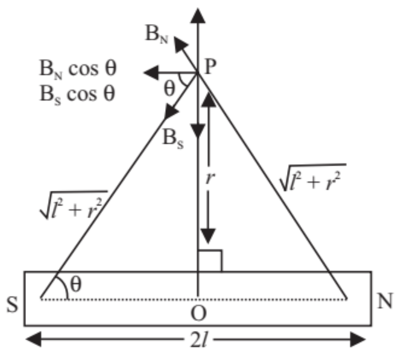
- **Magnetic Field on Equator of Bar Magnet:** Consider a bar magnet of length $2l$ and pole strength q_m.
  Point P lies on equatorial line of the magnet at a distance r from its centre.
  Force experienced per unit pole strength due to north pole

$$B_N = \frac{\mu_o}{4\pi}\frac{q_m}{PN^2} = \frac{\mu_o}{4\pi}\frac{q_m}{(l^2+r^2)} \text{ along NP}$$

Force exp. per unit pole strength due to south pole

$$B_S = \frac{\mu_0}{4\pi}\frac{q_m}{(PS)^2} = \frac{\mu_o}{4\pi}\frac{q_m}{(l^2+r^2)} \text{ along PS}$$

$$|\vec{B}_N| = |\vec{B}_S|$$



Vertical components of B_N and B_S are equal and opposite to each other so they cancel out.
Horizontal components will add up.

$$|\vec{B}_{net}| = B_N\cos\theta + B_S\cos\theta = \frac{2\mu_o}{4\pi}\frac{q_m}{(l^2+r^2)}\cos\theta$$

$$\cos\theta = \frac{l}{(l^2+r^2)^{\frac{1}{2}}}$$

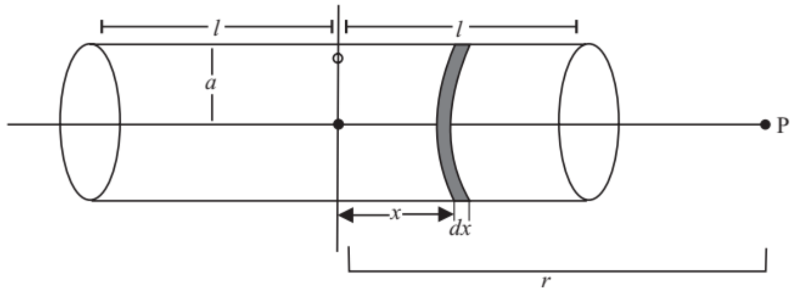
$$|\vec{B}_{net}| = \frac{2\mu_o}{4\pi}\frac{q_m}{(l^2+r^2)} \times \frac{l}{(l^2+r^2)^{\frac{1}{2}}} = \frac{\mu_o m}{4\pi(l^2+r^2)^{\frac{3}{2}}}$$

Direction of B is anti-parallel ($\theta = 180°$) to magnetic moment.

- Special cases: (*i*) If $r >>> l$

$$|\vec{B}| = \frac{\mu_o m}{4\pi(r^2)^{\frac{3}{2}}} = \frac{\mu_0 m}{4\pi r^3}$$

- **Show that Solenoid is Equivalent to a Bar Magnet:** Magnetic field lines of a bar magnet and solenoid show similar patterns. Let us calculate the axial field of a finite solenoid.
  Consider a solenoid consisting of 'n' terns per unit length. Let length of solenoid = $2l$ and radius = a. Let P be the point at a distance r from centre of solenoid where we want to find the field intensity.



Each turn of solenoid is at a different distance from the point P. Consider a circular element of thickness = dx of the solenoid at a distance x from the centre.
No. of turns in element = $n \times dx$
Magnetic field produced by these turns = dB.

$$dB = \frac{2\mu_o I \pi a^2 n\, dx}{4\pi [(r-x)^2 + a^2]^{\frac{3}{2}}}$$

Total magnetic field = $B = \int dB$

$$B = \frac{\mu_o n \cdot I \cdot a^2}{2} \int_{-l}^{+l} \frac{dx}{[(r-x)^2 + a^2]^{\frac{3}{2}}}$$

For the points that are very far away i.e., $r >>> l$ and $r >>> a$

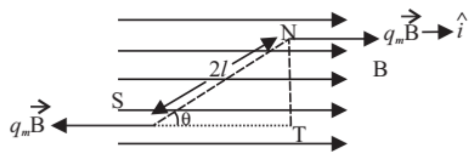
$$B = \frac{\mu_o n I a^2}{2r^3} \int_{-l}^{+l} dx = \frac{\mu_o n I a^2}{2r^3}[l - (-l)]$$

$$= \frac{\mu_o n I a^2 2l}{2r^3} = \frac{\mu_o n I l a^2}{r^3}$$

Multiply and divide by 4π, we get $= \frac{\mu_o}{4\pi} \frac{4\pi I n a^2 l}{r^3} = \frac{\mu_o}{4\pi} \frac{4 n I A l}{r^3}$ [n = turns per unit length]

$$B = \frac{\mu_o}{4\pi} \frac{2NIA}{r^3} = \frac{\mu_o}{4\pi} \frac{2m}{r^3} \quad [\because m = NIA], N = n \times 2l$$

- **Magnetic Dipole Placed in a Uniform Magnetic Field:** Consider a dipole of length $2l$ placed in a uniform magnetic field B. Let θ be the angle between dipole moment and B.



Force experienced by N-pole = $q_m \vec{B} \hat{i}$
Force experienced by S-pole = $q_m \vec{B}(-\hat{i})$
$\vec{F}_{net} = 0$

These forces will form a couple, which will rotate the dipole. The torque acting

$$\tau = |\vec{F}| \times \text{perpendicular distance between forces}$$

$$= q_m B \times NT = q_m B\, 2l \sin\theta = mB \sin\theta$$

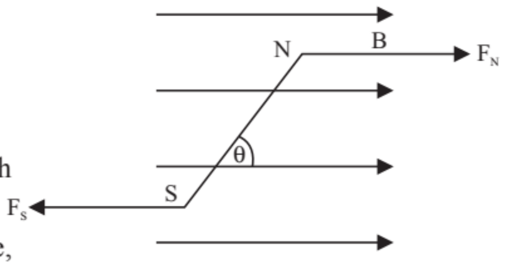
or $\vec{\tau} = \vec{m} \times \vec{B}$

Time period of oscillation of a dipole placed in a uniform magnetic field:

$T = 2\pi\sqrt{\frac{I}{mB}}$, where I is moment of inertia.

- **Potential Energy of Magnetic Dipole Placed in Uniform Magnetic Field**

Consider a dipole of length = $2l$ placed in uniform magnetic field B. Dipole moment makes angle θ with magnetic field.



There is a torque acting on it.

$$\tau = m\text{B}\sin\theta.$$

where m = dipole moment

B = Magnetic field strength

If this dipole is rotated by small angle $d\theta$ in anticlockwise direction, then work needs to be done,

$$dW = \tau d\theta = m\text{B}\sin\theta\, d\theta$$

$$W = \int dW = \int_{\theta_i}^{\theta_f} m\text{B}\sin\theta\, d\theta = m\text{B}\int_{\theta_i}^{\theta_f}\sin\theta\, d\theta$$

$$= m\text{B}\left[-\cos\theta\right]_{\theta_i}^{\theta_f} = m\text{B}[-\cos\theta_f + \cos\theta_i] = m\text{B}[\cos\theta_i - \cos\theta_f]$$

$\therefore$ $W = m\text{B}(\cos\theta_i - \cos\theta_f)$

This work done is stored as potential energy (U)

$$U = m\text{B}(\cos\theta_i - \cos\theta_f)$$

If $\theta_i = 90°$, $\theta_f = \theta$, $U = m\text{B}(\cos 90° - \cos\theta) = -m\text{B}\cos\theta = -\vec{m}\cdot\vec{\text{B}}$

For a magnetic dipole to be in equilibrium

Net force =0, Net torque = 0

i.e., $\theta = 0°$ or $\theta = 180°$

For stable equilibrium: F = 0, Torque = 0, potential energy should be minimum

So $\theta = 0°$. i.e., dipole moment should be parallel to the magnetic field.

For unstable equilibrium: F = 0, Torque = 0

Potential energy should be maximum

i.e., $\theta = 180°$. Dipole moment should be anti-parallel to the magnetic field.