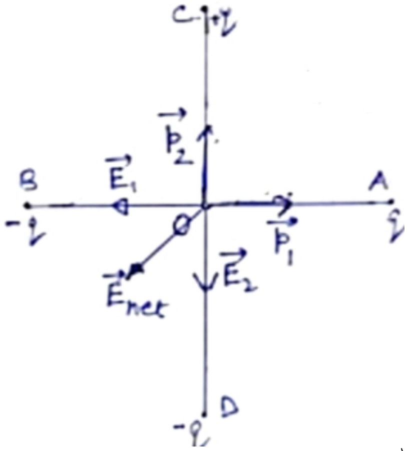
Solution

ELECTRIC FIELD AND CHARGES NUMERICALS

Class 12 - Physics

1. 

Dipole moment due to dipole BA is $\overrightarrow{p_1}$ & dipole moment due to dipole DC is $\overrightarrow{p_2}$.

Electric field $\overrightarrow{E_1}$ due to $\overrightarrow{p_1}$ is along OB.

Electric field $\overrightarrow{E_2}$ due to $\overrightarrow{p_2}$ is along OD.

Magnitude of resultant Electric field

$\left|\overrightarrow{E_{net}}\right| = 2\sqrt{2}E$

Since $\left|\overrightarrow{E_1}\right| = \left|\overrightarrow{E_2}\right| = E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2}$

$\vec{E}_{\text{net}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\sqrt{2}q}{a^2}$

Direction of $\overrightarrow{E_{\text{net}}}$ is 225^o to x-axis.

2. The two spheres will share the final charge equally. Let q be the charge on each sphere.

$\therefore$ F = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1q_2}{r^2}$ = 0.025 N

or $\frac{9\times10^9\times q\times q}{(0.90)^2}$ = 0.025

or $q^2 = \frac{0.025\times(0.90)^2}{9\times10^9} = 225 \times 10^{-14}$

or $q = 1.5 \times 10^{-6}$ C

3. a. $\phi = \vec{E} \cdot A\hat{i} = EA$

$= 3 \times 10^3 \times (100 \times 10^{-4})$

$= 30\ NC^{-1}m^2$

b. ϕ = EA cos 60^o

$= 15\ NC^{-1}m^2$

4. 

Due to induction by charge +Q at centre, the inner surface acquires -Q charge and outer surface of shell will acquire +Q charge. So,

i. Surface charge density on outside surface, $\sigma = \frac{+Q}{4\pi R_2^2}$

ii. Surface charge density on inside surface, $\sigma = \frac{-Q}{4\pi R_1^2}$

5. Here $q = \frac{1}{3}\times 10^{-7}$ C, 2a = 2 cm = 0.02 m,

$E = 3 \times 10^7\ NC^{-1}$

τ_{max} = pE sin 90° = q × 2a × E × 1

$= \frac{1}{3} \times 10^{-7} \times 0.02 \times 3 \times 10^{7} \times 1$ = 0.02 Nm

6. i. Here, $\phi = 8.0 \times 10^{3}$ Nm^{2}C^{-1}

   Suppose that the net charge inside the box is q.

   Then, according to Gauss theorem,

   $\phi = \frac{q}{\varepsilon_0}$

   or q = $\varepsilon_0 \phi$

   Now, $\varepsilon_0 = 8.854 \times 10^{-12}$ C^{2}N^{-1}m^{-2}

   $\therefore$ q = $8.854 \times 10^{-12} \times 8.0 \times 10^{3}$

   = 70.832×10^{-9} C

   ii. If the net outward flux through the surface of the box is zero, it cannot be concluded that there is no charge inside the box. There may be equal amounts of positive and negative charges inside the box. Therefore, if the net outward flux is zero, we cannot conclude that the charge inside the box is zero. One can only say that the net charge inside the box is zero.

7. Here q = 2.0×10^{-4} C, m = 10g = 10^{-2} kg

   r = 10 cm = 0.10 m

   Let v be the speed of each particle at infinite separation. By conservation of energy,

   P.E. of two particles at the separation of 10 cm = K.E. of the two particles at infinite separation

   $\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

   or $v^2 = \frac{1}{4\pi\varepsilon_0}, \frac{q_1,q_2}{rm}$

   $= \frac{9\times10^9\times2.0\times10^{-4}\times2.0\times10^{-4}}{0.10\times10^{-2}}$ = 36×10^{4}

   $\therefore$ v = 600 ms^{-1}

8. According to a principle of superposition of electric fields, E (electric field) at a point due to the system of three charges will be the superposition of all the electric field due to all the charges and can be calculated as:

   $E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \frac{q_3}{r_{3P}^2}\hat{r}_{3P}\right]$

   Here, r_1 = 1 m, r_2 = 3 m and r_3 = 9 m and so on.

   Thus, Electric field,

   $E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(1)^2} + \frac{1}{(3)^2} + \frac{1}{(9)^2} + \ldots + \infty\right]$

   Here all the terms are in G.P with common ratio $\frac{1}{9}$ which can be solved as-

   $E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1-1/9}\right]$

   $E = \frac{q}{4\pi\varepsilon_0} \times \frac{9}{8} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{9q}{8} NC^{-1}$

   This electric field will be directed away from the charges if charges are positive or towards them if they are negative.

9. a. Radius of sphere, $R = \frac{d}{2} = \frac{2.4}{2} = 1.2\text{m}$

   Surface charge density, $\sigma = 80 \times 10^{-6}\text{Cm}^{-2}$

   $\therefore \sigma = \frac{q}{4\pi R^2}$ or $q = 4\pi R^2 \sigma$

   Charge on the sphere is given by (q)

   $q = 4 \times 3.14 \times (1.2)^2 \times 80 \times 10^{-6}$

   $q = 1.447 \times 10^{-3}\text{C}$

   b. Gauss's law states that the net flux of an electric field in a closed surface is directly proportional to the enclosed electric charge.

   Electric flux is given by, $\phi = \frac{q}{\varepsilon_0}$

   $\phi = \frac{1.447\times10^{-3}}{8.85\times10^{-12}}$ = 1.63×10^{8} Nm^{2}C^{-1}

10. Here q = -2.0×10^{-6}C,

    σ = 4.0×10^{-6} Cm^{-2}

    Field produced by charged plate,

    E = $\frac{\sigma}{2\varepsilon_0}$

    Force of attraction between the charged particle and the plate,

$\therefore$ F = qE = $\frac{\sigma q}{2\varepsilon_0} = \frac{4\times10^{-6}\times2.0\times10^{-6}}{2\times8.85\times10^{-12}}$

= 0.45 N

11. a. i. Here, point O is equidistant from all the charges at the endpoint of the pentagon. Thus, due to symmetry, the forces due to all the charges are canceled out; As a result, the electric field at O is zero.When charge q is removed, it is equivalent to placing a -q charge at the same point. The electric field at point O would be $\vec{E} = \frac{q}{4\pi s_0 r^2}$ along $\overrightarrow{OA}$If charge q at A is replaced by -q, which is equivalent to placing two negative charges i.e., -2q at the same point. So the value of the electric field would be $\overrightarrow{E'} = \frac{2q}{4\pi\varepsilon_o r^2}$ along $\overrightarrow{OA}$

 b. If pentagon shown here is replaced by any n sided regular polygon with charge q at each of its corners, again charges are symmetrical about the centre thus the electric field at O would continue to be zero.

12. For the charge +Q situated at origin O, the field $\vec{E}$ points along +ve x-direction i.e., towards the right.

 a. The outward drawn normal on cap PSQ points towards the left while it points towards the right for caps PRQ, PWQ, and circle PTQ. So the Flux is negative for (i) and positive for rest.

 b. The same electric field lines crossing (i) also cross (ii), (iii). Also, by Gauss's law, the fluxes through (iii) and (iv) add up to zero. Hence, all magnitudes of fluxes are equal.

 c. Given area of the cap (ii) = A

 Electric field through cap (ii) is

 E = $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$ = $9 \times 10^9 \times \frac{Q}{(\sqrt{2})^2}$

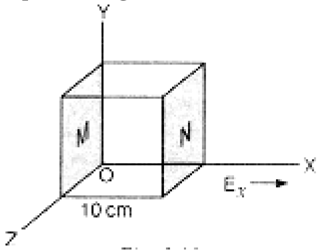
 = 4.5×10^9 Q NC^{-1}

 Electric flux through the cap (ii) is

 ϕ_E = EA

 = 4.5×10^9 QA NC^{-1} m^2

13. i. Since the electric field has X-component only, the angle θ between the direction of electric field and the area vectors representing the four faces of the cube (except the faces M and N shown shaded) is $\pi/2$ in each case.



 Therefore, electric flux through each of these four faces,

 $\phi = E_x S \cos\theta = E_x S \cos \pi/2 = 0$

 The electric field varies along X-axis as given by

 E_x = 5 A x + 2 B

 Therefore, magnitude of electric field at face M (x = 0),

 E_M = 5 A x + 2 B = $5 \times 10 \times 0 + 2 \times 5 = 10$ NC^{-1}

 and magnitude of electric field at face N (x = 0.1 m),

 E_N = 5 A x + 2B = $5 \times 10 \times 0{\cdot}1 + 2 \times 5 = 15$ NC^{-1}

 The electric flux through the face M,

 $\phi_M = E_M S \cos\theta = 10 \times (0{\cdot}1 \times 0{\cdot}1) \cos 0° = 0{\cdot}10$ NmC^{-1} (inwards)

 The electric flux through the face N,

 $\phi_N = E_N S \cos\theta = 15 \times (0{\cdot}1 \times 0{\cdot}1) \cos 0° = 0{\cdot}15$ NmC^{-1} (outwards)

 Therefore, net electric flux through the cube,

 $\phi = \phi_N - \phi_M = 0{\cdot}15 - 0{\cdot}10 = 0{\cdot}05$ NmC^{-1}

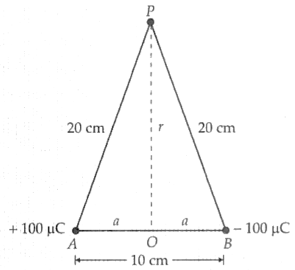
 ii. Now, from Gauss' theorem, we have

 $q = \varepsilon_0\phi$

 = $8{\cdot}854 \times 10^{-12} \times 0{\cdot}05 = 4{\cdot}43 \times 10^{-13}$ C

14. Here q = 100 μC = 10^{-4}C, 2a = 10 cm = 0.10 m

p = q $\times$ 2a = $10^{-4} \times 0.10 = 10^{-5}$ Cm



Clearly,

$(r^2 + a^2)^{1/2}$ = 20 cm = 0.20 m

$E_{equa} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2+a^2)^{3/2}}$

$= \frac{9\times10^9\times10^{-5}}{(0.2)^3} = \frac{9}{8}\times\ 10^7$

= 1.125 $\times$ 10^7 NC^{-1}

15. Electric field due to a line charge at distance r from it,

$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$

Force exerted by this field on charge q,

$F = eE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q\lambda}{r}$

Force exerted on negative charge (r = 0.02 m),

$F_1 = \frac{9\times10^9\times2\times2\times10^{-8}\times4\times10^{-4}}{0.02}$ N

= 7.2 N, acting towards the ling charge

Force exerted on positive charge (r = 2.2 $\times 10^{-2}$ m),

$F_2 = \frac{9\times10^9\times2\times2\times10^{-8}\times4\times10^{-4}}{2.2\times10^{-2}}$

= 6.5 N, acting away from the line charge

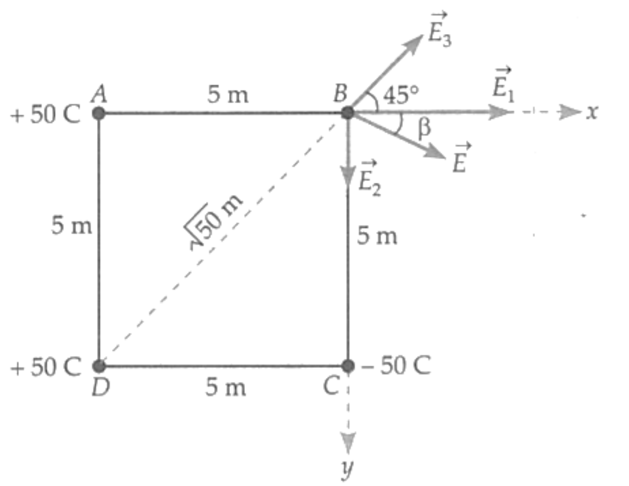
Net force on the dipole,

$F = F_1 - F_2$ = 7.2 - 6.5

= 0.7 N, acting towards the line charge.

16. Electric field at B due to + 50 C charge at A is

$E_1 = k \cdot \frac{q}{r^2} = k \cdot \frac{50}{5^2}$ = 2k, along AB



Electric field at B due to -50 C charge at C is

$E_2 = k \cdot \frac{50}{5^2}$ = 2k, along BC

Electric field at B due to + 50 C charge at D is

$E_3 = k \cdot \frac{50}{\left(\sqrt{5^2+5^2}\right)^2}$ = k, along DB

Component of E_1 along x-axis = 2 k (as it acts along x-axis)

Component of E_2 along x-axis = 0 (as it acts along y-axis)

Component of E_3 along x-axis

= $E_3 \cos 45^o = k \cdot \frac{1}{\sqrt{2}} = \frac{k}{\sqrt{2}}$

$\therefore$ Total electric field at B along x-axis

$E_x = 2k + 0 + \frac{k}{\sqrt{2}} = k\left(2 + \frac{1}{\sqrt{2}}\right)$

Now,

Component of E_1 along x-axis = 0

Component of E_2 along y-axis = 2k

Component of E_3 along y-axis

$E_y = E_3 \sin 45^o = k \cdot \frac{1}{\sqrt{2}} = \frac{k}{\sqrt{2}}$

But the components of E_2 and E_3 act in opposite directions, therefore, total electric field at B along y-axis

$= 2k - \frac{k}{\sqrt{2}} = k\left(2 - \frac{1}{\sqrt{2}}\right)$

$\therefore$ Resultant electric field at B will be

$E = \sqrt{E_x^2 + E_y^2}$

$= \sqrt{\left[k\left(2+\frac{1}{\sqrt{2}}\right)\right]^2 + \left[k\left(2-\frac{1}{\sqrt{2}}\right)\right]^2} = \sqrt{9k^2}$

$= 3k = 3 \times 9 \times 10^9 \ NC^{-1} = 2.7 \times 10^{10} \ NC^{-1}$

If the resultant field E makes angle β with x-axis, then

$\tan\beta = \frac{E_y}{E_x} = \frac{(2-1/\sqrt{2})k}{(2+1/\sqrt{2})k} = 0.4776$ or $\beta = 25.5^o$

17. a. Consider an electric dipole placed in a uniform electric field of strength E in such a way that its dipole moment $\vec{p}$ makes an angle θ with the direction of $\vec{E}$. The charges of dipole are -q and +q at separation 2l the dipole moment of electric dipole,

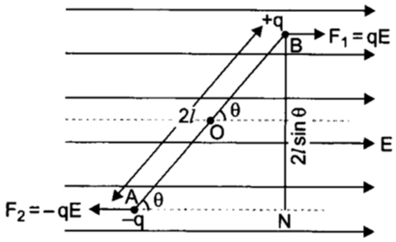
p = q·2l ...(i)

Force: The force on charge +q is, $\vec{F}_1 = q\vec{E}$, along the direction of field $\vec{E}$.

The force on charge -q is $\vec{F}_2 = qE$, opposite to the direction of field $\vec{E}$.

Obviously forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude but opposite in direction; hence net force on electric dipole in uniform electric field is

$F = F_1 - F_2 = qE - qE = 0$ (zero)



As net force on electric dipole is zero, so dipole does not undergo any translatory motion.

Torque: The forces $\vec{F}_1$ and $\vec{F}_2$ form a couple (or torque) which tends to rotate and align the dipole along the direction of electric field. This couple is called the torque and is denoted by τ.

$\therefore$ Torque τ = magnitude of one force $\times$ perpendicular distance between lines of action of forces

= qE (BN) = qE(2l sin θ) = (q2l) E sin θ

= pE sin θ [using (i)] ...(ii)

Clearly, the magnitude of torque depends on orientation (θ) of the electric dipole relative to electric field. Torque (τ) is a vector quantity whose direction is perpendicular to the plane containing $\vec{p}$ and $\vec{E}$ given by right hand screw rule.

In vector form $\vec{\tau} = \vec{p} \times \vec{E}$ ...(iii)

Thus, if an electric dipole is placed in an electric field in oblique orientation, it experiences no force but experiences a torque. The torque tends to align the dipole moment along the direction of electric field.

Maximum Torque: For maximum torque sin θ should be the maximum. As the maximum value of sin θ = 1 when $\theta = 90^\circ$

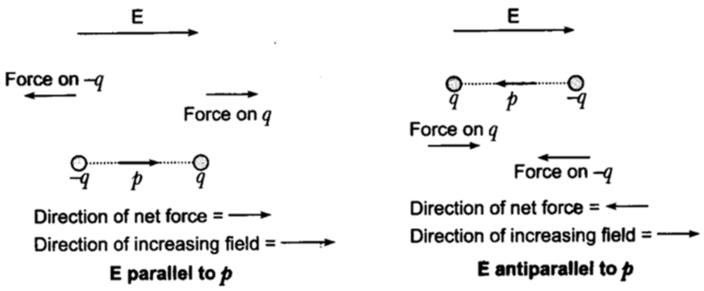
$\therefore$ Maximum torque, $\tau_{\max} = pE$

When the field is non-uniform, the net force will evidently be non-zero. There will be translatory motion of the dipole.

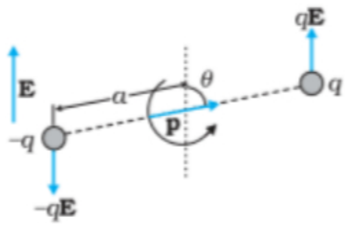
When $\vec{E}$ is parallel to $\vec{p}$, the dipole has a net force in the direction of increasing field.

When $\vec{E}$ is anti-parallel to $\vec{p}$, the net force on the dipole is in the direction of decreasing field.

In general, force depends on the orientation of $\vec{p}$ with respect to $\vec{E}$.



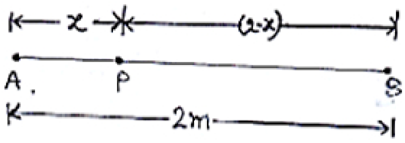
b. Let an electric dipole be rotated in electric field from angle θ_0 to θ_1 in the direction of electric field. In this process the angle of orientation θ is changing continuously; hence the torque also changes continuously. Let at any time, the angle between dipole moment $\vec{p}$ and electric field $\vec{E}$ be θ then
Torque on dipole τ = pE sin θ
The work done in rotating the dipole a further by small angle dθ is
dW = Torque $\times$ angular displacement = pE sin θ dθ
Total work done in rotating the dipole from angle θ_0 to θ_1 is given by
$W = \int_{\theta_0}^{\theta_1} pE\sin\theta d\theta = pE[-\cos\theta]_{\theta_0}^{\theta_1}$
= -pE[cos $\theta_1 - \theta_0$] = pE(cos θ_0 - cos θ_1) ...(i)
Special case : If electric dipole is initially in a stable equilibrium position ($\theta_0 = 0^\circ$) and rotated through an angle $\theta\,(\theta_1 = \theta)$ then work done
W = pE[cos 0^o - cos θ] = pE(1 - cos θ) ...(ii)

18. a. 

From diagram shown above
Magnitude of Torque = (qE)(2a sinθ)
= (2qa) (E sinθ)
= pE sinθ
For direction $\vec{\tau} = \vec{p} \times \vec{E}$

i. for maximum Torque, dipole should be placed perpendicular to the direction of electric field
θ = 90^o = $\frac{\pi}{2}$

ii. For the torque to be half the maximum,
θ = 30^o = $\frac{\pi}{6}$

b. 

The electric field at P point should be equal and opposite, therefore E_{PA} = E_{PB} ; E = $\frac{kq}{r^2}$
$\frac{kq_A}{x^2} = \frac{kq_B}{(2-x)^2}$
$\frac{1}{x^2} = \frac{4}{2-x)^2}$
$\frac{1}{x} = \frac{2}{2-x}$
x = $\frac{2}{3}m$ therefore, at distance 2/3 m from point A, the electric field should be zero.