Solution

SAITECHINFO CENTUM CYCLIC UNIT TEST | ELECTRIC FIELD AND CHARGES | CLASS 12 PHYSICS

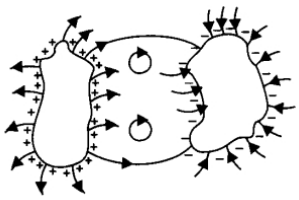
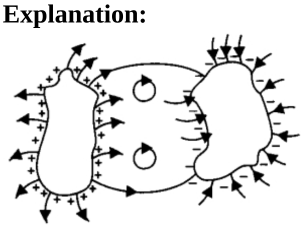
Class 12 - Physics

1.
   (b) 2E
   Explanation:
   Electric field due to the point charge = E = $\frac{K\times 2q}{r^2}$
   Electric field due to the spherical shell = E' = $\frac{K\times q}{(r/2)^2}$ = 2E

2.
   (c) Zero
   Explanation:
   On all the dipoles, net charge = 0. Hence net charge enclosed within the surface = 0. So the total electric flux coming out of the surface, $\phi = \frac{q_{net}}{\varepsilon_0} = 0$

3.
   (c) $\frac{\vec{F}}{8}$
   Explanation:
   The electric field at a distance r from the dipole is $\overrightarrow{E} = K\frac{2P}{r^3}$, so $E \propto \frac{1}{r^3}$
   Force on charge q is $F = qE$ also $F \propto \frac{1}{r^3}$
   If distance r is doubled, then force will $F' = \frac{F}{8}$

4. **(a)** 3.0×10^4 Vm^{-1}
   Explanation:
   Given: Electric dipole moment,
   $p = 2 \times 10^{-8}\ N/C$
   Maximum torque, $\tau = 6 \times 10^{-4}\ N - m$
   $\theta = 90^\circ$
   Electric field, $E =$?
   The torque acting on the dipole:
   $\tau = pE\sin\theta$
   $\Rightarrow 6 \times 10^{-4} = 2 \times 10^{-8} \times E$
   $\Rightarrow E = 3 \times 10^4 Vm^{-1}$

5.
   (c) 
   Explanation:
   

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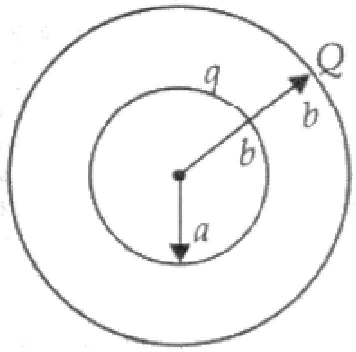
6. 

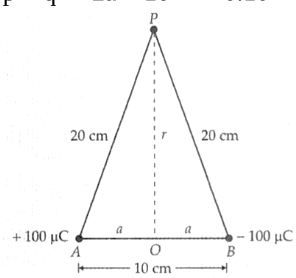
Field due to a uniformly charged spherical shell is zero at all points inside the shell. So in the given question, we have:

i. For 0 < x < a
The point lies inside both the spherical shells.
Hence, E(x) = 0

ii. For $a \leq x < b$
Point is outside the spherical shell of radius 'a' but inside the spherical shell of radius 'b'
$\therefore E(x) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}$

iii. $b \leq x < \infty$
Point is outside of both the spherical shells. Total effective charge at the centre equals (Q + q)
$\therefore E(x) = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q+Q}{x^2}\right)$

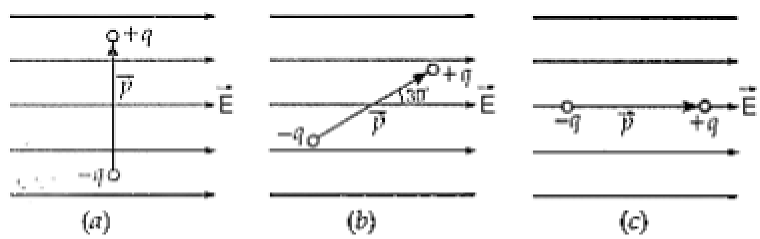
7. Here q = 100 μC = 10^{-4}C, 2a = 10 cm = 0.10 m
p = q $\times$ 2a = $10^{-4} \times$ 0.10 = 10^{-5} Cm



Clearly,
$(r^2 + a^2)^{1/2}$ = 20 cm = 0.20 m
$E_{equa} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2+a^2)^{3/2}}$
$= \frac{9\times10^9\times10^{-5}}{(0.2)^3} = \frac{9}{8}\times\ 10^7$
= 1.125 $\times$ 10^7 NC^{-1}

8. When an electric dipole is placed in a uniform electric field, the torque acting on the dipole is given by
$\vec{\tau} = \vec{p} \times \vec{E}$
In the above expression, one pair of perpendicular vectors is $\vec{\tau}$ and $\vec{p}$ the other is $\vec{\tau}$ and $\vec{E}$.
The magnitude of the torque on the dipole is given by τ = p E sin θ

i. It follows that the torque will be maximum (= p E), when the dipole is placed perpendicular to the direction of the electric field as shown in Fig. (a).



ii. For the torque to be half the maximum value,
p E sin θ = p E/2
or sin θ = 1/2
or θ = 30°

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Therefore, torque on the dipole will be half the maximum value, when it is placed making an angle of 30° to the direction of the electric field as shown in Fig. (b).

iii. It follows that the torque will be zero when the dipole is placed along the direction of the electric field as shown in Fig. (c).

9. The velocity of the particle, normal to the direction of field

v_x = 1000 ms^{-1}, is constant

The velocity of the particle, along the direction of field, after 10 s, is given by

$v_y = u_y + a_y t$

$= 0 + \frac{qE_y}{m}t = \frac{2\times10^{-6}\times10^{3}\times10}{10\times10^{-6}}$ = 2000 ms^{-1}

The net velocity after 10 s,

v = $\sqrt{v_x^2 + v_y^2} = \sqrt{(1000)^2 + (2000)^2}$ = 1000 $\sqrt{5}$ ms^{-1}

Displacement, along the x-axis, after 10 s,

x = 1000 $\times$ 10m = 10000 m

Displacement along y-axis (in the direction of field) after 10 s,

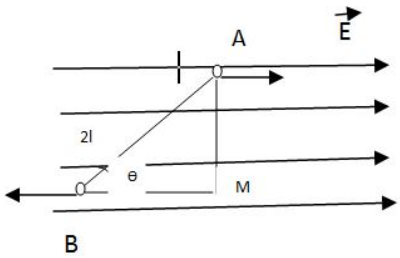
y = $u_y t + \frac{1}{2}a_y t^2$ = (0)t + $\frac{1}{2}\frac{qE_y}{m}t^2 = \frac{1}{2} \times \frac{2\times10^{-6}\times10^{3}}{10\times10^{-6}}\times$ (10)^2

= 10000 m

Net displacement,

r = $\sqrt{x^2 + y^2} = \sqrt{(10000)^2 + (10000)^2}$ = 10000 $\sqrt{2}$ m

10. Consider a dipole of length 2l is placed at angle θ with the direction of uniform electric field E. Force acts on positive and negative charges in the direction of field and opposite to the field direction as shown in the diagram.



Magnitude of force on the charges at A and B are given by

F = qE

As the forces are equal and opposite, so net force is zero but due to different lines of action these produce torque.

Torque = Force $\times$ perpendicular distance

$\tau = qE \times AM$

Now, $AM = \ AB\ \sin\theta = 2l\ \sin\theta$

$\tau = qE\ \times 2l\ \sin\theta\ =\ pE\ \sin\theta$

Two perpendicular vectors are force vector and component of dipole moment along AM.

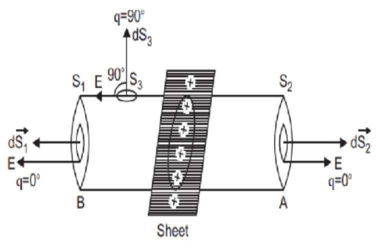
11. Gauss Theorem: The net outward electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

$\oint \overrightarrow{E}\,.\overrightarrow{dS} = \frac{1}{\varepsilon_0}\sum q$

Electric field due to a uniformly charged infinite plane sheet:

Let electric charge be uniformly distributed over the surface of a thin, non-conducting infinite sheet. Consider a cylindrical Gaussian surface having two plane faces A and B lying on the opposite sides and parallel to the charged sheet and the cylindrical surface perpendicular to the sheet as shown in figure.

By symmetry, the electric field strength at every point on the flat surface is the same and its direction is normal outwards at the points on the two plane surfaces and parallel to the curved surface.



Total electric flux,

$\oint E.dS = \int_{S_1}\overrightarrow{E}\,.\overrightarrow{dS_1} + \int_{S_2}\overrightarrow{E}\,.\overrightarrow{dS_2} + \int_{S_3}\overrightarrow{E}\,.\overrightarrow{dS_3}$

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$\oint E.dS = \int_{S_1} E.dS_1 \cos\ 0 + \int_{S_2} E\ dS_2 \cos\ 0 + \int_{S_3} EdS_3 \cos 90$

$\oint E.dS = E\int_{S_1} dS_1 + E\int_{S_2} dS_2 + 0$

$\oint E.dS = 2E\int_{S_1} dS_1 = 2Ea$ ...... (i)

If σ is charge per unit area of sheet and 'a' is the intersecting area,

the charge enclosed by Gaussian surface = σa

According to Gauss’s theorem

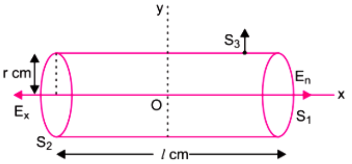
Total electric flux $= \oint E.dS = \frac{1}{\varepsilon_0}q$

$\oint E.dS = \frac{1}{\varepsilon_0}\sigma a$ ..... (ii)

From (i) and (ii) we have,

$E = \frac{\sigma}{2\varepsilon_0}$

Thus electric field strength due to an infinite flat sheet of charge is independent of the distance of the point and is directed normally away from the charge.

12. 

$\varphi_1 = \oint_{S_1} \vec{E}_1 \cdot \overrightarrow{dS}_1 = \oint_{S_1} \left(E_x \hat{i}\right) \cdot \left(dS_1 \hat{i}\right) = E_x S_1$

Electric flux through flat surface S_2

$\varphi_1 = \int_{S_2} \vec{E}_2 \cdot \overrightarrow{dS}_2 = \int_{S_2} \left(-E_x \hat{i}\right) \cdot \left(-dS_2 \hat{i}\right)$

$= \int_{S_2} E_x dS_2 = E_x S_2$

Electric flux through curved surface S_3

$\varphi_3 = \int_{S_3} \left(\vec{E}_3 \cdot \overrightarrow{dS_3}\right) = \int_{S_3} E_3 dS_3\ \cos 90^o = 0$

$\therefore$ Net electric flux, $\varphi = \varphi_1 + \varphi_2 = E_x(S_1 + S_2)$

But $S_1 = S_2 = \pi\left(r \times 10^{-2}\right)^2\ m^2 = \pi r^2 \times 10^{-4}\ m^2$

$\therefore \varphi = E_x .2\ (\pi r^2 \times 10^{-4})$ units

By Gauss's law, $\varphi = \frac{1}{\varepsilon_0}q$

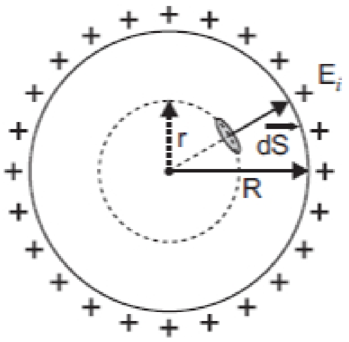
$q = \varepsilon_0 \varphi = \varepsilon_0 E_x \left(2\pi r^2 \times 10^{-4}\right)$

$= 2\pi\varepsilon_0 E_x r^2 \times 10^{-4} = 4\pi\varepsilon_0 \left(\frac{E_x r^2 \times 10^{-4}}{2}\right)$

$= \frac{1}{9\times10^9}\left[\frac{E_x r^2 \times 10^{-4}}{2}\right]$

$= 5.56\ E_x r^2 \times 10^{-11}$ coulomb.

13. a. According to Gauss theorem, the electric flux through a closed surface depends only on the net charge enclosed by the surface and not upon the shape or size of the surface. For any closed arbitrary shape of the surface enclosing a charge the outward flux is the same as that due to a spherical Gaussian surface enclosing the same charge. Justification: This is due to the fact that (i) Electric field is radial and (ii) The electric field $E \propto 1/R^2$ Thus, electric field at each point inside a charged thin spherical shell is zero.

b. 

$= \int_S \overrightarrow{\mathbf{E}}_i \cdot d\overrightarrow{\mathbf{S}}$

$= \int E_i\ dS\ \cos 0 = E_i \cdot 4\pi r^2$

Now, Gaussian surface is inside the given charged shell, so charge enclosed by Gaussian surface is zero. Hence, by Gauss’s

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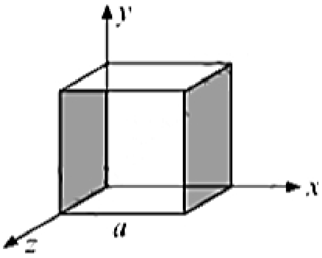
theorem

$\int_S \overrightarrow{\mathbf{E}_i} \cdot \overrightarrow{d\mathbf{S}} = \frac{1}{\varepsilon_0} \times$ charge enclosed

$\Rightarrow E_i 4\pi r^2 = \frac{1}{\varepsilon_0} \times 0 \Rightarrow E_i = 0$

Thus, electric field at each point inside a charged thin spherical shell is zero.

c. Since the electric field has only x component, for faces normal to x direction, the angle between E and ΔS is $\pm\frac{\pi}{2}$. Therefore, the flux is separately zero for each face of the cube except the two shaded ones.



The magnitude of the electric field at the left face is E_L = 0 (As x = 0 at the left face) The magnitude of the electric field at the right face is E_R = 2a (As x = a at the right face) The corresponding fluxes are

right face is E_r = 2a

(As x = a at the right face)

The corresponding fluxes are

$\phi_L = \vec{E} \cdot \Delta\vec{S} = 0$

$\phi_R = \vec{E}_R \cdot \Delta\vec{S}$

$= E_R \Delta S \cos\theta$

$= E_R \Delta S\,(\because \theta = 0^\circ)$

$\Rightarrow \phi R = ERa^2$

Net flux (ϕ) throught the cube $= \phi\mathrm{L} + \phi\mathrm{R}$

= 0 + ERa^2 = ERa^2

ϕ = $2a(a)^2 = 2a^3$

We can use Gauss's law to find the total charge q inside the cube.

$\phi = \frac{q}{\varepsilon_0}$

$\mathrm{q} = \phi\varepsilon_0 = 2\mathrm{a}^3\varepsilon_0$

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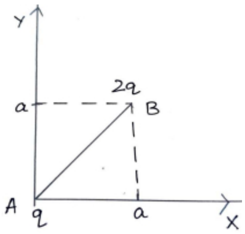
14. i. Force between two point charges varies inversely with the square of distance between the charges and is directly proportional to the product of magnitude of the two charges and acts along the line joining the two charges.

$\overrightarrow{F_{12}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}{}^2}\widehat{r_{12}}$

Where $\overrightarrow{r_{12}}$ is a vector from charge q_2 to charge q_1.

ii. In derivation of Gauss's law, flux is calculated using Coulomb's law and surface area. Here coulomb's law involves $\frac{1}{r^2}$ factor and surface area involves r^2 factor. When product is taken, the two factors cancel out and flux becomes independent of r.

iii. 

$\vec{r} = \overrightarrow{AB} = a\hat{i} + a\hat{j}$

$\mathrm{r} = |\overrightarrow{AB}| = \sqrt{a^2 + a^2} = \sqrt{2}a$

$\vec{F} = \frac{1}{4\pi\in_o}\frac{q_1 q_2}{r^2}\hat{r}$

$\vec{F} = \frac{1}{4\pi\in_o} \times \frac{q \times 2q}{(\sqrt{2}a)^2} \times \frac{(a\hat{i} + a\hat{j})}{\sqrt{2}a}$

$\vec{F} = \frac{1}{4\pi\epsilon_o} \times \frac{2q^2}{2a^2} \times \frac{(\hat{i} + \hat{j})}{\sqrt{2}}$

$\vec{F} = \frac{1}{4\pi\in_o} \times \frac{q^2}{\sqrt{2}a^2} \times (\hat{i} + \hat{j})$

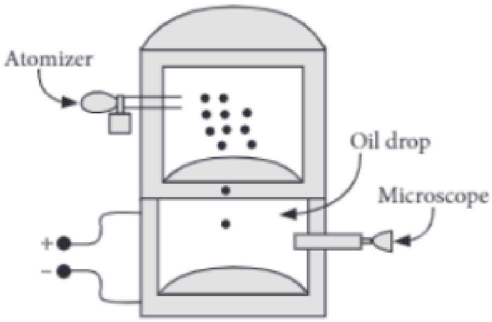
$\vec{F} = \frac{q^2}{4\sqrt{2}\pi\in_o a^2}(\hat{i} + \hat{j})$

$|\vec{F}| = \frac{1}{4\pi\in_o}\frac{q^2}{a^2}$

15. **Read the text carefully and answer the questions:**

In 1909, Robert Millikan was the first to find the charge of an electron in his now-famous oil-drop experiment. In that experiment, tiny oil drops were sprayed into a uniform electric field between a horizontal pair of oppositely charged plates. The drops were observed with a magnifying eyepiece, and the electric field was adjusted so that the upward force on some negatively charged oil drops was just sufficient to balance the downward force of gravity. That is, when suspended, upward force qE just equaled Mg.

Millikan accurately measured the charges on many oil drops and found the values to be whole number multiples of 1.6×10^{-19} C the charge of the electron. For this, he won the Nobel prize.



(i) **(a)** 6.40×10^{-19} C

Explanation:

As, qE = mg ⇒ q = $\frac{1.08\times10^{-14}\times9.8}{1.68\times10^{5}}$ = 6.4×10^{-19} C

(ii) **(a)** 4

Explanation:

q = ne or ⇒ n = $\frac{6.4\times10^{-19}}{1.6\times10^{-19}}$ = 4

(iii) **(c)** 10^{12}

Explanation:

For the drop to be stationary,

Force on the drop due to electric field = Weight of the drop

qE = mg

q = $\frac{mg}{E} = \frac{1.6\times10^{-6}\times10}{100}$ = 1.6×10^{-7} C

Number of electrons carried by the drop is

n = $\frac{q}{e} = \frac{1.6\times10^{-7}\text{C}}{1.6\times10^{-19}\text{C}}$ = 10^{12}

(iv) **(b)** charge is quantized

Explanation:

charge is quantized

(v) **(d)** 4μC

Explanation:

Millikan's experiment confirmed that the charges are quantized, i.e., charges are small integer multiples of the base value which is charge on electron. The charges on the drops are found to be multiple of 4. Hence, the quanta of charge is 4 μC.

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