Electricity

Lecture Notes: Current Electricity

3.13 Kirchhoff’s Rules

Kirchhoff's Current Law (KCL):

- **Statement**: At any junction (node) in an electrical circuit, the sum of currents flowing into the junction is equal to the sum of currents flowing out of the junction.
  - **Mathematical Expression**: $\sum I_{\text{in}} = \sum I_{\text{out}}$
  - This law is based on the principle of conservation of electric charge.

Kirchhoff's Voltage Law (KVL):

- **Statement**: The sum of the electromotive forces (emf) and potential differences (voltage drops) in any closed loop in a circuit is equal to zero.
  - **Mathematical Expression**: $\sum \text{emf} + \sum \text{voltage drops} = 0$
  - This law is based on the principle of conservation of energy.

Application of Kirchhoff’s Rules in Complex Circuits:

- **Steps to Apply Kirchhoff’s Rules**:
  1. **Label All Currents**: Assign a current to each branch of the circuit and indicate its direction.
  2. **Apply KCL**: At each junction, apply Kirchhoff's Current Law to write equations for the sum of currents.
  3. **Apply KVL**: For each independent loop in the circuit, apply Kirchhoff's Voltage Law to write equations for the sum of voltages.
  4. **Solve the Equations**: Use the equations from KCL and KVL to solve for the unknown currents and voltages in the circuit.

Example Problems and Solutions:

Example 1: Simple Circuit with Two Loops

- **Circuit Description**: A circuit with two loops, one containing a 12V battery and two resistors $R_1 = 4\Omega$ and $R_2 = 6\Omega$, and the other loop containing a 9V battery and one resistor $R_3 = 3\Omega$.

- **Solution**:

  1. **Label Currents**: Let I_1 be the current in the first loop (clockwise) and I_2 be the current in the second loop (clockwise).
  2. **Apply KCL**: At the junction where R_1, R_2, and R_3 meet, I_1 splits into I_2 and the current through R_3. Therefore, $I_1 = I_2 + I_3$.
  3. **Apply KVL to Loop 1**:
     - $12V - I_1R_1 - (I_1 - I_2)R_2 = 0$

    - $12V - 4\Omega I_1 - 6\Omega(I_1 - I_2) = 0$
    - $12V - 4\Omega I_1 - 6\Omega I_1 + 6\Omega I_2 = 0$
    - $12 = 10I_1 - 6I_2$ (Equation 1)
4. **Apply KVL to Loop 2**:
    - $9V - I_2R_3 - (I_2 - I_1)R_2 = 0$
    - $9V - 3\Omega I_2 - 6\Omega(I_2 - I_1) = 0$
    - $9V - 3\Omega I_2 - 6\Omega I_2 + 6\Omega I_1 = 0$
    - $9 = -9I_2 + 6I_1$ (Equation 2)
5. **Solve the Equations**:
    - From Equation 2: $6I_1 - 9I_2 = 9 \Rightarrow 2I_1 - 3I_2 = 3$ (Equation 3)
    - From Equation 1: $10I_1 - 6I_2 = 12 \Rightarrow 5I_1 - 3I_2 = 6$ (Equation 4)
    - Subtract Equation 3 from Equation 4:
        - $(5I_1 - 3I_2) - (2I_1 - 3I_2) = 6 - 3$
        - $3I_1 = 3 \Rightarrow I_1 = 1A$
    - Substitute $I_1 = 1A$ into Equation 3:
        - $2(1A) - 3I_2 = 3 \Rightarrow 2 - 3I_2 = 3$
        - $-3I_2 = 1 \Rightarrow I_2 = -1/3A$

Example 2: Complex Circuit with Three Loops

- **Circuit Description**: A circuit with three loops containing batteries and resistors of various values.

- **Solution**:

1. **Label Currents**: Assign currents I_1, I_2, and I_3 to each branch.
2. **Apply KCL at Each Junction**:
    - Write current equations for each junction.
3. **Apply KVL to Each Loop**:
    - Write voltage equations for each loop.
4. **Solve the System of Equations**:
    - Use simultaneous equations to find the values of I_1, I_2, and I_3.