

Solution

RELATIONS AND FUNCTIONS -MCQ

Class 12 - Mathematics

1. (a) Reflexive and symmetric but not transitive

Explanation:

Reflexivity: clearly for every $a \in S$ (real number)

$$|a - a| = 0 < 1$$

$\Rightarrow R$ is Reflexive

Symmetric: $(a, b) \in R$

$$\Rightarrow |a - b| < 1$$

$$\Rightarrow |b - a| < 1$$

$$\Rightarrow (b, a) \in R$$

$\therefore R$ is Symmetric

Transitive: $(a, b), (b, c) \in R$

$$\Rightarrow |a - b| < 1 \text{ and } |b - c| < 1$$

$$\text{Now, } |a - c| = |a - b + b - c| \leq |a - b| + |b - c|$$

Which need not be less than 1

So, R is not Transitive

$\therefore R$ is Reflexive and symmetric but not transitive.

- 2.

- (b) transitive but neither reflexive nor symmetric

Explanation:

Since, x is greater than $y \forall x, y \in \mathbb{N}$

Let $(x, x) \in R$

For $x \in \mathbb{N}, x > x$ is not true for any $x \in \mathbb{N}$.

Therefore, R is not reflexive.

Let $(x, y) \in R \Rightarrow xRy$

$$x > y$$

but $y > x$ is not true for any $x, y \in \mathbb{N}$

Thus, R is not symmetric.

Let xRy and yRz

$$x > y \text{ and } y > z$$

$$\Rightarrow x > z$$

$$\Rightarrow xRz$$

So, R is transitive.

- 3.

- (c) (1, 2)

Explanation:

By removing (1, 2), we get identity relation. We know that identity relation is always an equivalence relation.

- 4.

- (c) $xRy, x \leq y$

Explanation:

If R is a relation defined by xRy : if $x \leq y$, then R is reflexive and transitive. But, it is not symmetric. Hence, R is not an equivalence relation.

- 5.

- (c) $aRb \Leftrightarrow a < b$

Explanation:

$a R b$ if $a < b$

Let $a \in \mathbb{R}$ and a be an integer here.

Therefore, $a < a$, we can see that it is not possible.

It is not satisfying the condition, therefore, we can say that the given relation is not reflexive.

Now, we will check if the relation is symmetric or not.

Let $a R b$ if $a < b$

We cannot write $a < b$ as $b < a$ i.e.,

$$\Rightarrow a < b \neq b < a$$

Hence, the given relation is not symmetric.

Now, we will check if the relation is transitive or not.

Let $a R b$ and $b R c$ if $a < b$ and $b < c$ is an even integer.

On adding both these, we get

$$\Rightarrow a + b < b + c$$

On further simplification, we get

$$\Rightarrow a < c$$

We can say that $b R c$

Therefore, the given relation is also transitive.

As the given relation is only transitive, therefore, the given relation is not an equivalence relation.

6.

(c) Reflexive, transitive but not symmetric

Explanation:

Since n divides n , $\forall n \in \mathbb{N}$, R is reflexive.

R is not symmetric since for $(3, 6) \in R$ but $(6, 3) \notin R$.

R is transitive since for n, m, r whenever n/m and $m/r \Rightarrow n/r$, i.e., n divides m and m divides r , then n will divide r .

7.

(c) $f(x) = x + 2$

Explanation:

Injectivity: Let $x, y \in \mathbb{Z}$, then, $f(x) = f(y) \Rightarrow x + 2 = y + 2 \Rightarrow x = y \Rightarrow f$ is one-one.

Surjectivity: Let $f(x) = y$, where $y \in \mathbb{Z}$, $\Rightarrow x + 2 = y \Rightarrow x = y - 2 \in \mathbb{Z}$, $\Rightarrow f$ is onto.

Therefore, f is a bijective function.

8.

(b) many-one

Explanation:

The given function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 + x$.

$$f(x) = x^2 + x$$

Now, for $x = 0$ and -1 , we have

$$f(0) = 0 \text{ and } f(-1) = 0$$

Hence, $f(0) = f(-1)$ but $0 \neq -1$

$\Rightarrow f$ is not one one $\Rightarrow f$ is many one.

9. (a) many-one and into

Explanation:

Given that $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = [x]^2 + [x + 1] - 3$

As $[x]$ is the greatest integer so for different values of x , we will get same value of $f(x)$.

$[x]^2 + [x + 1]$ will always be an integer.

So, f is many-one.

Similarly, in this function co domain is mapped with at most one element of domain because for every $x \in \mathbb{R}$, $f(x) \in \mathbb{Z}$.

So, f is into.

10.

(d) range of $f = Y$

Explanation:

A function $f : A \rightarrow B$ is said to be onto, if for every $b \in B$, there exists an element a in A such that $f(a) = b$.

11. Let $f(x) = 6x, x \in \mathbb{N}$

For one - one

$$f(x_1) = 6x_1$$

$$f(x_2) = 6x_2$$

put $f(x_1) = f(x_2)$ we get

$$6x_1 = 6x_2$$

Hence, if $f(x_1) = f(x_2), x_1 = x_2$

Function f is one - one

For onto

$$f(x) = 6x$$

let $f(x) = y$, such that $y \in \mathbb{N}$

$$6x = y$$

$$\Rightarrow x = \frac{y}{6}$$

If $y = 1$

$$x = 0.166667$$

which is not possible as $x \in \mathbb{N}$

Hence, f is not onto.

12. Since the school is a boys school, no student of the school can be the sister of any student of the school. Hence, $R = \phi$, showing that R is the empty relation.

It is also obvious that the difference between the heights of any two students of the school has to be less than 3 meters. This shows that $R' = A \times A$ is the universal relation.

13. Let $f(x) = x + 1$

$$\therefore g(x) = \begin{cases} x - 1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

These are two examples in which $g \circ f$ is onto but f is not onto.

14. Here, $f(-1) = f(1) = 1$ so that distinct elements have same image. Therefore, f is not one-one.

Clearly, f takes positive real values, so $\text{range}(f) = \mathbb{R}^+ \neq \mathbb{R} = \text{codomain}$.

So f is not onto

15. $(a, a) \in R$

16. $(3, 1)$ is the single ordered pair which needs to be added to R to make it the smallest equivalence relation.

17. Injectivity: We know that $f(0) = \cos 0 = 1$ and $f(2\pi) = \cos 2\pi = 1$.

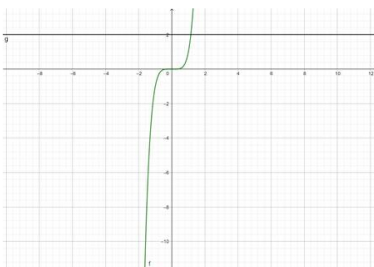
So, different elements in R (domain) may have the same image. Hence, it is not an injection.

Surjectivity: Since the range of $\cos x$ is -1 to 1 , it follows that the range of $f(x)$ is not equal to its co-domain. So, f is not surjective.

18. one - one and onto

$$f(x) = x^5$$

$$\Rightarrow y = x^5$$



Since the lines do not cut the curve in 2 equal valued points of y , therefore, the function $f(x)$ is one - one.

The range of $f(x) = (-\infty, \infty) = \mathbb{R}$ (Codomain)

$\therefore f(x)$ is onto

$\therefore f(x)$ is one - one and onto.

19. Set of all elements related to 1 is $\{1\}$.

20. Let $x_1, x_2 \in \mathbb{R}$ be such that $f(x_1) = f(x_2)$

$$\Rightarrow x_1^3 = x_2^3$$

$$\Rightarrow x_1 = x_2$$

Therefore, f is one-one function, hence $f(x) = x^3$ is injective.

21.

(d) A is false but R is true.

Explanation:

Assertion: The elements that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is $\{1\}$.

Reason: Since, R_1 and R_2 are equivalence relations, therefore $(a, a) \in R_1, (a, a) \in R_2, \forall a \in A$.

This implies that $(a, a) \in R_1 \cap R_2, \forall a$.

Hence, $R_1 \cap R_2$ is reflexive.

Further, $(a, b) \in R_1 \cap R_2 \Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2$ and $(b, a) \in R_2$

$$\Rightarrow (b, a) \in R_1 \cap R_2$$

Hence, $R_1 \cap R_2$ is symmetric.

Similarly, $(a, b) \in R_1 \cap R_2$ and $(b, c) \in R_1 \cap R_2$

$$\Rightarrow (a, c) \in R_1 \text{ and } (a, c) \in R_2 \Rightarrow (a, c) \in R_1 \cap R_2.$$

This implies that $R_1 \cap R_2$ is transitive.

Hence, $R_1 \cap R_2$ is an equivalence relation.

22.

(d) A is false but R is true.

Explanation:

Assertion: Let $A = \{1, 2, 3, 4, 5, 6\}$

A relation R is defined on set A is

$$R = \{(a, b) : b = a + 1\}$$

$$\therefore R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$$

Now, $6 \in A$ but $(6, 6) \notin R$

$\therefore R$ is not reflexive.

Reason: Given set $A = \{1, 2, 3\}$

A relation R on A is defined as

$$R = \{(1, 2), (2, 1)\}$$

$$\therefore (1, 2) \in R \text{ and } (2, 1) \in R.$$

So, R is symmetric.

23.

(d) A is false but R is true.

Explanation:

We have, $A = \{1, 5, 8, 9\}$, $B = \{4, 6\}$ and $f = \{(1, 4), (5, 6), (8, 4), (9, 6)\}$

So, all elements of B has a domain element on A or we can say elements 1 and 8 & 5 and 9 have some range 4 & 6, respectively.

Therefore, $f : A \rightarrow B$ is a surjective function not one to one function.

Also, for a bijective function, f must be both one to one onto.

24. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion Here, $R = \{(x, y) : y \text{ is divisible by } x\}$ is a relation in the set $A = \{1, 2, 3, 4, 5, 6\}$.

For reflexive, we know that x is divisible by x , which is true for all $x \in A$.

$\therefore (x, x) \in R$ for all $x \in A$.

So, R is reflexive.

For symmetry, we observe that 6 is divisible by 2 i.e. $(2, 6) \in R$ but 2 is not divisible by 6 i.e. $(6, 2) \notin R$.

So, R is not symmetric.

For transitivity, let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow y$ is divisible by x and z is divisible by y .

$\Rightarrow z$ is divisible by x .

$\Rightarrow (x, z) \in R$

e.g. 2 is divisible by 1 and 4 is divisible by 2.

So, 4 is divisible by 1. So, R is transitive.

Hence, R is not an equivalence relation.

25.

(c) A is true but R is false.

Explanation:

Assertion: It is given that $f : R^* \rightarrow R^*$ is defined by

$$f(x) = \frac{1}{x}$$

For one-one, $f(x) = f(y)$

$$\Rightarrow \frac{1}{x} = \frac{1}{y}$$

$$\Rightarrow x = y$$

Therefore, f is one-one.

For onto, it is clear that for $y \in R^*$, there exists $x = \frac{1}{y} \in R^*$ (exists as $y \neq 0$) such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$$

Therefore, f is onto. Thus, the given function (f) is one-one and onto.

Reason: Now, consider function $g : N \rightarrow R^*$ defined by $g(x) = \frac{1}{x}$.

$$\text{We have, } g(x_1) = g(x_2) \Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$$

$$\Rightarrow x_1 = x_2$$

Therefore, g is one-one.

Further, it is clear that g is not onto as for $1 \cdot 2 \in R^*$, there does not exist any x in N such that $g(x) = 1 \cdot 2$

$$\Rightarrow \frac{1}{x} = 1 \cdot 2$$

$$\Rightarrow x = \frac{1}{1 \cdot 2} \notin N \text{ (domain)}$$

Hence, function g is one-one but not onto.

26. (a) Both A and R are true and R is the correct explanation of A .

Explanation:

Given $A = \{0, 1\}$

$$f(2n - 1) = 0, f(2n) = 1 \quad \forall n \in N$$

\Rightarrow every element in A has its preimage in N .

so A is true.

and we know range is subset or equal to codomain

so R is true.

and for onto function, Range = Codomain

so R is correct explanation of A .

27. (a) Both A and R are true and R is the correct explanation of A .

Explanation:

Given, $n(A) = 4$

Total number of reflexive relations = $2^{n(n-1)}$

$$= 2^{4 \times 3} = 2^{12}$$

28.

(d) A is false but R is true.

Explanation:

The assertion is false because relation R is not symmetric, $(1, 2) \in R$ but $(2, 1) \notin R$

The reason is true because for a relationship to be equivalence it must be reflexive, symmetric, and transitive.

29. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: Given that,

$A = \{2, 4, 6\}$,

$R = \{3, 5, 7, 9\}$

and $R = \{(2,3), (4, 5), (6, 7)\}$

Here, $f(2) = 3$, $f(4) = 5$ and $f(6) = 7$

It can be seen that the images of distinct elements of A under f are distinct.

Hence, function f is one-one but f is not onto as $9 \in B$ does not have a pre-image in A.

Hence, both Assertion and Reason are true, but Reason is not a correct explanation of Assertion.

30.

(d) A is false but R is true.

Explanation:

Assertion is false because every function is not invertible. The function which is one-one and onto i.e. bijective functions are invertible so reason is true.

31. State whether the given statement is True or False:

(i) **(a)** True

Explanation: {

True

(ii) **(a)** True

Explanation: {

True

(iii) **(a)** True

Explanation: {

True

Given that, $f : \mathbb{N} \rightarrow A$ defined by $f(2n-1) = 0$, $f(2n) = 1, \forall n \in \mathbb{N}$,

We observe that for each element of A (co-domain) there exist a pre image in \mathbb{N} (domain).

For $0 \in A$ there exist $(2n-1), \forall n \in \mathbb{N}$

For $1 \in A$ there exist $(2n), \forall n \in \mathbb{N}$

Thus, the mapping is onto.

(iv) **(a)** True

Explanation: {

True

(v) **(b)** False

Explanation: {

False

(vi) **(b)** False

Explanation: {

False

Given that, $R = \{(3, 1), (1, 3), (3, 3)\}$ be a relation defined on the set $A = \{1, 2, 3\}$

Now,

R is not reflexive $\because (1,1), (2,2) \notin R$.

R is symmetric $\because (3,1) \in R \Rightarrow (1,3) \in R$

R is not transitive $\because (1,3) \in R$ and $(3,1) \in R$ but $(1,1) \notin R$.

(vii) **(a)** True

Explanation: {

True

(viii) **(b)** False

Explanation: {

False

Given that, $A = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$

Now,

R is not reflexive $\because (2,2) \notin R$

(ix) **(a)** True

Explanation: {

True

(x) **(a)** True

Explanation: {

True

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